

**B.A. Vorontsov-
Velyaminov**

**A COLLECTION
OF PROBLEMS
IN
ASTRONOMY**

The Mir Titles Project

A Collection of Problems in Astronomy: A Manual for Students

B.A. Vorontsov-Vel'yaminov

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СБОРНИК ЗАДАЧ ПО АСТРОНОМИИ

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Collection of Problems in Astronomy

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Front cover image: A close-up shot of part of the Tarantula
Nebula. Credit: NASA, ESA. [[link](#)]

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Preface

This collection of problems is intended for secondary school students. The topics of the problems and their arrangement correspond to the syllabus and the textbook.

A distinctive feature of this problem book is that it includes assignments for independent work. Independent observations and measurements based on drawings and photographs simulate the work of an astronomer and give students an idea of how science studies objects. To increase the number of such exercises and make fuller use of the illustrative material in the astronomy textbook, some of the independent assignments are proposed to be carried out using its drawings and photographs.

When solving the problems, it is sometimes necessary to use a movable star chart and tabular data, which are also provided in the textbook.

The answers given at the end of the book help to check the correctness of problem solutions.

Translator's Preface

It was a good exercise to translate the book and in the process refresh my basic astronomy concepts. The book has some good problems which requires some careful thinking on part of the learner.

Several of the locations used are from erstwhile USSR and Russia, but the problems can be readily adapted to other locations. Since this problem book uses refers to several images from the textbook, I have included them as a part of the text. This makes the book self-contained. In the future we might also translate the two-part textbook itself.

Though I have taken care while translating, there might be errors in the translation, please let us know if you find any.

I hope this the English translation of this problem book helps in understanding basic astronmy and associated methods of problems solving for young astronomers.

Damitr Mazanav



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1 Introduction

1.1 Independent Observations of the Sky to Become Acquainted with the Subjects Covered in the Textbook

When studying the astronomy course, independent observations of the starry sky are absolutely essential. Only in this way will you gain a good understanding of the fundamentals of the science of the sky.

1. On the very first clear evening, go out under the evening sky, if possible to a place with an open view of it. Find a location where streetlights and light from windows do not interfere with seeing the stars. Locate the constellation of Ursa Major, and from it find the Pole Star, using [Figure 1](#) (Figure 4 of the textbook). Determine the points of the compass and remember their positions in relation to landmarks on the

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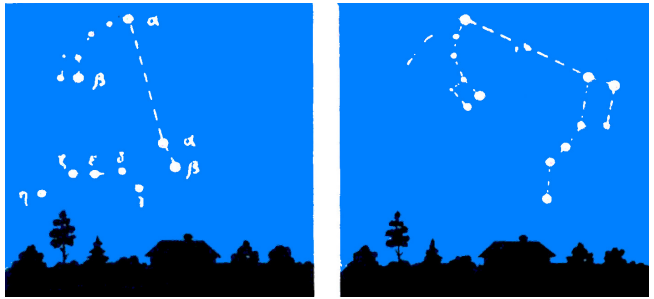


Figure 1: The change in the position of the constellations Ursa Major and Ursa Minor relative to the horizon during the diurnal rotation of the sky. (Figure 4 from textbook)

- ground. Draw the position of Ursa Major in relation to the horizon. After 1–2 hours, mark its new position on the same drawing. Find also the constellations Cassiopeia, Lyra, Cygnus, and Aquila (the Summer Triangle).
2. Observe a close group of stars, for example the constellation Lyra, the constellation Delphinus, or the star cluster the Pleiades. Compare how many stars are visible to the naked eye and through binoculars. Estimate approximately the field of view of your binoculars, bearing in mind that the distance between the stars forming the front edge of the bowl of Ursa Major is 5° .
- Instruction:* For carrying out Task 2 you will need

1.1 Independent Observations of the Sky

binoculars. Adjust them for your eyes so that the stars appear as points.

3. First, measure with a simple clinometer the altitude above the horizon of the Pole Star, and then of several bright stars, the names of which you should determine using a star chart. Record the results and show them to your teacher.

Instruction: For carrying out Task 3 you will need a simple clinometer, which can easily be made yourself from a protractor, fixing a plumb line at its centre and a sight, visible in dim light, at the ends of the base (for example, made from thick white paper – see [Figure 2](#)).

4. The length of an arc (and approximately also the chord) subtending an angle of 1° is 57 times shorter than the radius. The angular diameter of the Moon and the Sun is 0.5° . Therefore, the Moon and the Sun can be covered by a paper disc of 0.5 cm placed at a distance of 60 cm from the eye. Verify this.

Note: The Sun may be observed only through very dark photographic film or smoked glass.

5. Using the *School Astronomical Calendar* and a planisphere, locate in the evening sky the main constellations and planets. Draw their positions in relation to the nearest stars, even if the latter are faint. Repeat this observation after 1–2 months.

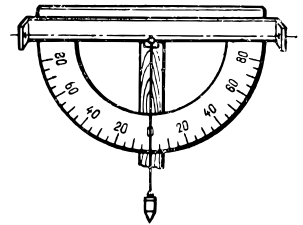


Figure 2: A simple homemade clinometer.

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Compare the results obtained and explain them.

6. Look at several of the brightest stars and compare their colours. Record your colour estimates. Check your results using Table IV given at the end of the textbook.

Note: Through binoculars, differences in colour are seen more clearly.

7. The magnitudes of the stars of Ursa Major are: α and β – 1.8 and 2.4 respectively. By comparing with them, estimate approximately the brightness of the other stars in the same constellation.

Note: The designations of the stars in the constellation Ursa Major are given in [Figure 1](#) (Figure 4 of the textbook).

8. By comparing two parts of [Figure 1](#) in the textbook, estimate how much time has passed between the two marked positions of Ursa Major, assuming the drawings were made during the same night.
9. Estimate approximately the geographical latitude of the place for which [Figure 1](#) in the textbook was made, given that the distance from the Pole Star to the star ζ (zeta) of Ursa Major is 34° .
10. Place a small object, such as a matchbox, on the surface of a table at one of its corners. Move to the opposite end of the table and, using a protractor, measure the two angles formed by the short side

1.1 Independent Observations of the Sky

of the table and the lines connecting the table's corners with the object. Using the two measured angles and the length of the short side of the table, draw a triangle. Graphically (or trigonometrically) determine the distance from the table's corners to the given object (or the length of the table). Then measure the length of the table directly and compare the results.

Instruction: Task 10 is more convenient to carry out on a long table. In it, you perform the same angular measurements that astronomers make in the sky using a telescope and precisely graduated circles – vertical and horizontal.

Note: Parallax is the angle at an object under which the baseline (perpendicular to the line of sight) is seen from the object. Parallax equals 180° minus the sum of the angles adjacent to the ends of the baseline and formed by the baseline and the straight lines connecting the ends of the baseline with the object. The experiment described is a determination of distance by measuring parallax.

11. Using the method described in Task 10, practise determining the distances to various objects by measuring their parallax.
12. Measure angle A in Figure 36 of the textbook (Figure 3 here) with a protractor, and angle $\angle ASC$ in

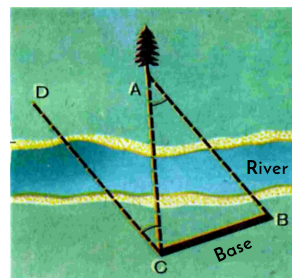


Figure 3: Measurement of the distance to an inaccessible object by parallax displacement.

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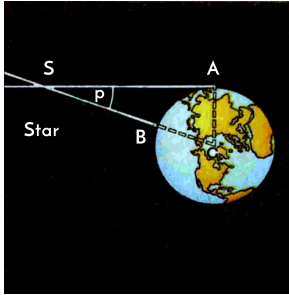


Figure 4: The horizontal parallax of a star (celestial body).

Figure 37 (Figure 4 here). Using a ruler, measure the lengths of the baselines. Calculate from these respectively the distances CA and SC , and verify the results by direct measurements on the drawings.

Measure the angles p and ρ on Figure 38 (Figure 5 here) using a protractor, and determine from the obtained data the ratio of the diameters of the depicted bodies.

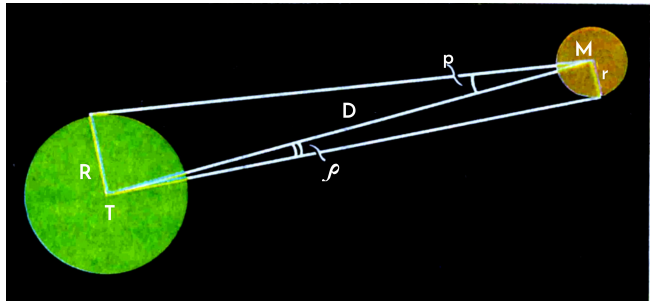


Figure 5: Determination of the linear dimensions of celestial bodies from their angular dimensions.

1.2 Brightness of Celestial Bodies and Stellar Magnitudes

The proposed problems will help to understand the concept of the stellar magnitude scale. This old term, “stellar magnitude”, does not express the size of stars, but their brightness – the relative amount of light energy received from a star.

It is agreed that a star of the magnitude 1 is 2.512 times brighter than a star of the magnitude 2; a star of the 2 magnitude is 2.512 times brighter than a star of the 3 magnitude, and so on.

Consequently, a star of magnitude 1 is brighter than a star of magnitude 4 by a factor of

$$2.512 \cdot 2.512 \cdot 2.512 = 2.512^3 = 2.512^{4-1} \approx 15.85.$$

The brightnesses I_1 and I_2 of stars, and their magnitudes m_1 and m_2 , are related by the formulas:

$$I_1 : I_2 = 2.512^{m_2 - m_1} \quad \text{or} \\ \log(I_1 : I_2) = 0.4(m_2 - m_1).$$

The solution of the problems proposed below is based on the use of these relationships.

13. The faintest stars that can be recorded photographically with the largest telescope in the world (the

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- Soviet telescope with a mirror diameter of 6 m, installed on the Caucasus) are stars of magnitude 24. By what factor are they fainter than stars of magnitude 1?
14. A star's brightness varies by 7 magnitudes from minimum to maximum. By what factor does its brightness change?
 15. If we could bring a star of magnitude 4 twice as close, by what factor and by how many magnitudes would it appear brighter?
 16. What number of stars of magnitude 6, magnitude 5, magnitude 4 and magnitude 3 would produce as much light, in total, as one star of magnitude 1?
 17. There are 2000 stars of magnitude 6 in the northern sky. How many such stars would be required for their combined emission to equal the visible emission of Sirius ($m = -1.6$)?
 18. Two very closely spaced stars of equal brightness are seen in a telescope, but to the unaided eye they appear merged into one. By how many magnitudes does their combined brightness differ from the magnitude of one of them?

1.3 Celestial Sphere and Angular Measurements

19. In autumn, a hunter walks towards the North Star before dawn to reach the forest and returns after sunrise. How should he walk back, using the position of the Sun as a guide?
20. Is there a difference between the North Celestial Pole and the geographic North Pole?
21. Is there any place on Earth where a blindfolded person, upon starting to walk, will inevitably head north?
22. When (approximately) does a star rise if, a month earlier, it rose at 10 p.m.?
23. At what distance from the eye should a coin (diameter 1.7 cm) be placed so that it just covers the Moon?
24. A spherical balloon has a diameter of 13 m. How far away is the balloon if its angular diameter is half that of the Moon?
25. Determine the linear distance between two stars that are at distances r_1 and r_2 from us and appear in the sky separated by an angular distance θ .
26. Which observations prove that, during the Earth's annual motion, its axis of rotation does not change

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direction in space?

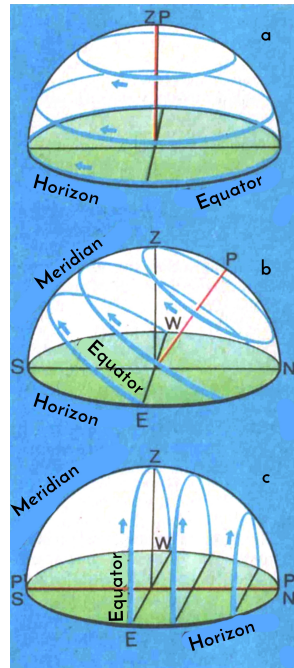


Figure 6: The diurnal trajectories of the stars relative to the horizon for an observer located at: a — the Earth's pole; b — the middle geographical latitudes; c — the equator.

1.4 Geographical Latitude, Visibility of Celestial Objects, Culminations, Celestial Coordinates, and Star Charts

For many of the questions in this section, it is sufficient to use the diagrams of the celestial sphere given in Figures 13 (Figure 6 here) and Figure 14 (Figure 7 here) of the textbook, and Figure 8 in this problem book, which shows the celestial sphere projected onto the plane of the meridian.

27. The latitude of Moscow is $\varphi = 55^\circ 45'$. Determine the angular distance from the zenith in Moscow to the celestial pole.
28. The southernmost regions of the former Soviet Union extend to a latitude of 35° . What angle does the plane of the celestial equator make with the horizon at that latitude?
29. At what angle does the celestial equator intersect the horizon (at the points of rising and setting) for an observer at latitude 40° ? How do these angles change if the latitude of the observation site is 10° , 20° , 50° , 70° , or -40° ?
30. In which two cases does the altitude of a celestial object above the horizon remain unchanged

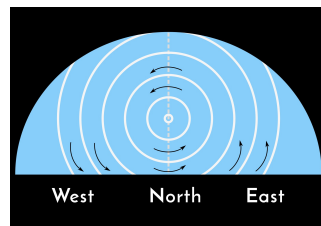


Figure 7: The visible diurnal paths of celestial bodies relative to the horizon in the northern part of the sky.

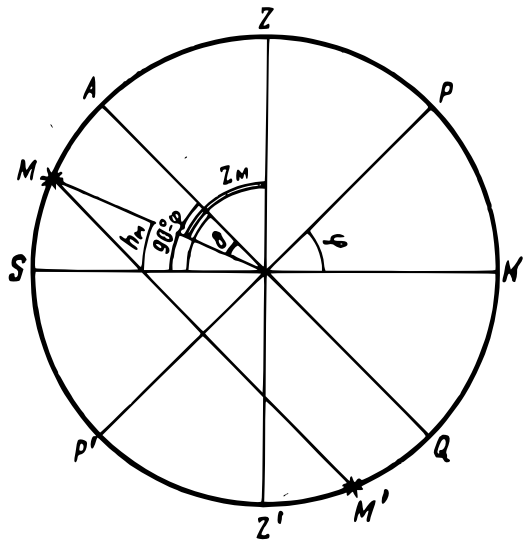


Figure 8: The celestial sphere projected onto the plane of the meridian. SN : horizon line, AQ : celestial equator, PP' : axis of the world (celestial axis), ZZ' : plumb line, Z – zenith, Z' – nadir, MM' : diurnal parallel of an arbitrary star M . The star M in this case has a negative (southern) declination δ ; h is its altitude.

throughout the day?

31. Using the instruments installed at the Soviet drifting station “North Pole”, it was possible, by observing stars, to determine its location on the globe with an accuracy of 250 m. This precision made it possible to track the drift of the ice floe. Determine the accuracy with which the geographical latitude

1.4 Geographical Latitude and Celestial Coordinates

was measured (the length of 1° of a meridian arc is 111 km).

32. What is the declination of stars that culminate in the zenith for a location whose geographical latitude is φ ?
33. What condition must the declination δ of a star satisfy for it to be circumpolar at geographical latitude φ ? What condition must it satisfy for it to be never rising?
34. What is the declination of stars that, at upper culmination, pass through the zenith of Moscow, Kiev and Tbilisi? (The geographical latitudes of the cities are, respectively, $\varphi = 55^\circ 45'$; $\varphi = 50^\circ 27'$; $\varphi = 41^\circ 43'$.)
35. What limiting declination allows stars of the southern hemisphere to be visible in Leningrad ($\varphi = 59^\circ 57'$) and in Tashkent ($\varphi = 41^\circ 20'$)?
36. In Moscow ($\varphi = 55^\circ 45'$) at noon the altitude of the Sun was $57^\circ 17'$. Using the astronomical calendar, determine on which day of the year this measurement was made.
37. State the declination of a star that can sometimes be observed on the horizon from any point on Earth.
38. What is the declination of a star that was observed

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in Arkhangelsk ($\varphi = 64^{\circ}32'$) at lower culmination at an altitude of 10° ?

39. Show that the altitude of a star at lower culmination is given by

$$h = \varphi + \delta - 90^{\circ},$$

where φ is the observer's geographical latitude and δ the star's declination.

40. What is the zenith distance of Vega (declination $\delta = +38^{\circ}42'$) during its upper culmination in Moscow ($\varphi = 55^{\circ}45'$)?
41. At what zenith distance does the upper culmination of the star Capella (declination $\delta = 45^{\circ}54'$) occur in Leningrad ($\varphi = 59^{\circ}57'$)? In Tashkent ($\varphi = 41^{\circ}20'$)?
42. In Odessa ($\varphi = 46^{\circ}29'$) the upper culmination of Sirius was observed at a zenith distance of $63^{\circ}5'$. What is its declination?
43. Derive the formula for the declination of stars that culminate at the north point for a location with geographical latitude φ .
44. Someone claimed that his acquaintance, living in Saratov ($\varphi = 51^{\circ}32'$), saw the star Capella (declination $\delta = 45^{\circ}54'$) during the day from a very deep well. Could this have happened? Explain your answer.

1.5 Ecliptic And Wandering Bodies

45. Using a planisphere, find those zodiacal constellations that lie above the horizon and are visible on a winter evening, and those that are visible on a spring evening.
46. To obtain a clear idea of how people could have plotted the position of the ecliptic on a star chart, measure the Sun's noontime altitude with a simple clinometer at least once a month. The results will show how the Sun shifts relative to the celestial equator. For greater clarity, plot these altitudes on a graph, remembering that the Sun shifts eastward by 30° each month.
47. How can you tell from the appearance of the starry sky and its rotation that you have arrived at the Southern Pole of the Earth?
48. How are the diurnal paths of stars situated with respect to the horizon for an observer living in the USSR? How do they differ from the diurnal paths of stars seen from the terrestrial equator?
49. A star's polar distance is $20^\circ 15'$. What is its zenith distance at lower culmination in Vologda ($\varphi = 59^\circ 13'$)?
50. A star lies 48° from the celestial pole. Is it always visible above the horizon at Odessa ($\varphi = 46^\circ 29'$),

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Moscow ($\varphi = 55^{\circ}45'$), Kiev ($\varphi = 50^{\circ}27'$) and Tbilisi ($\varphi = 41^{\circ}43'$)?

51. Does Fomalhaut (of *Piscis Austrinus*), whose declination is $\delta = -30^{\circ}5'$, rise at Arkhangelsk ($\varphi = 64^{\circ}35'$)?
52. Determine at which places on Earth (moving southwards) the constellation of the Southern Cross, whose stars have declinations in the range $\delta = -55^{\circ}$ to $\delta = -64^{\circ}$, begins to become visible. Trace on a globe or on hemispheric maps which countries contain these places.
53. At noon the length of the shadow of a vertical rod was equal to one third of its height. Calculate the geographical latitude of the place and give an approximate date of the observation, knowing that it was made in spring. (The Sun's declination was $\delta = +14^{\circ}47'$.)
54. Between which points of the horizon does the Sun rise and set on the days of the summer and winter solstices?
55. Using a model of the celestial sphere, determine how many times per year the Sun passes through the zenith for observers located on the equator and on the tropics.
56. On which days of the year does the Sun reach the zenith for an observer on the terrestrial equator?

1.5 *Ecliptic And Wandering Bodies*

57. What portion of the ecliptic remains permanently above the horizon?
58. Using the “School Astronomical Calendar”, determine the equatorial coordinates α and δ of the Sun on 5 May and on 22 October, and identify on a star chart the constellations in which the Sun lies on those dates.
59. Calculate the altitude of the Sun above the horizon at noon in Kazan and in Yerevan on the day of the winter solstice.
60. Using the “School Astronomical Calendar”, calculate the altitude of the Sun above the horizon at the moment of upper culmination on 31 May in Baku and in Gorky.
61. On what geographical latitude will the Sun culminate at noon at an altitude of 45° , if on that day its declination is $\delta = -10^\circ$?
62. List the zodiacal constellations that are visible above the horizon at midnight in our latitudes: (a) on the day of the vernal equinox; (b) on the day of the autumnal equinox; (c) on the days of each solstice.
63. Determine the geographical latitude at which the Sun culminates at the zenith on the day of the summer solstice.
64. Determine the geographical latitude of the place

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where, on the day of the winter solstice, the Sun's culmination occurs at the south point.

65. Using the “School Astronomical Calendar”, find the dates when the Sun becomes circumpolar and when the polar night begins at Hammerfest ($\varphi = 71^\circ$).
66. The altitude of the Sun at culmination on the day of the summer solstice was $73^\circ 27'$ (to the south of the zenith), and on the day of the winter solstice it was $26^\circ 33'$. Determine the geographical latitude of the observation site and the tilt of the ecliptic to the equator.
67. Around 1100 BCE Chinese astronomers found that on the day of the summer solstice the Sun's noon altitude was $79^\circ 7'$ (to the south of the zenith), and on the day of the winter solstice it was $31^\circ 19'$. Calculate the geographical latitude of the observation site and the angle of the ecliptic's tilt to the equator at that time.
68. The Pole Star is 58° from the celestial pole. What is its declination?
69. What are the azimuths of the north, south, east and west points?
70. What is the declination of the zenith at geographical latitude $\varphi = 42^\circ$?
71. What is the distance from the zenith to the celestial

1.5 *Ecliptic And Wandering Bodies*

pole at a location whose geographical latitude is $\varphi = +23^\circ 27'$?

72. Express 8 h 15 min 1 s in angular measure. Do the same for 13 h 20 min, 14 h 10 min, 20 h 50 min and 23 h 56 min.
73. Determine the right ascension and declination of the north pole of the ecliptic, the ecliptic being a great circle on the celestial sphere.
74. Determine the zenith distance of the Sun when the length of the shadow of a vertical object equals its height.
75. A celestial object has declination $\delta = +30^\circ$ and right ascension $\alpha = 7^h$. In which constellation is the object located? (See the star chart.)
76. What are the right ascension and declination of the vernal equinox point?
77. For which point on the celestial sphere are both right ascension and declination equal to zero?
78. In which constellations are the stars whose equatorial coordinates are:

a) $\alpha = 4^h 33^m$, $\delta = +16^\circ 25'$;

b) $\alpha = 16^h 26^m$, $\delta = -26^\circ 19'$;

c) $\alpha = 20^h 40^m$, $\delta = +45^\circ 6'$?

What are the proper names of these stars?

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79. Using the star chart, determine the equatorial coordinates (α, δ) of the stars Arcturus, Betelgeuse, Altair, Capella, Castor and Spica. Compare your results with the coordinates given in the textbook appendix (see Table IV).
80. Using the “School Astronomical Calendar”, take the equatorial coordinates of Jupiter on 16 April and on 1 October, and use a planisphere to determine its times of rising and setting. Is Jupiter observable on these dates from your locality?
81. Convert the right ascensions $284^{\circ}15'17''$, $170^{\circ}57'1''$, $191^{\circ}13'59''$ into hours, minutes and seconds of time.
82. Express the right ascensions $3^{\text{h}}17^{\text{m}}9^{\text{s}}$, $19^{\text{h}}27^{\text{m}}39^{\text{s}}$ and $21^{\text{h}}0^{\text{m}}3^{\text{s}}$ in degree measure.
83. Express in degrees the angle between the circles of declination of two stars, one of which culminated at $5^{\text{h}}12^{\text{m}}$ and the other at $5^{\text{h}}32^{\text{m}}$ of sidereal time.
84. A celestial object’s azimuth is 45° and its altitude is 60° . In which part of the sky should you look for it?
85. Two stars have the same right ascension. On what geographical latitude do both these stars rise and set simultaneously?
86. Using a planisphere, determine the date when the star with right ascension $\alpha = 18^{\text{h}}$ crosses the

1.5 *Ecliptic And Wandering Bodies*

meridian in the southern sky at 20^{h} local time in your locality.

87. From the chart, find the equatorial coordinates of the bright stars of the constellations Cygnus and Orion.
88. A planet crossed the meridian $2^{\text{m}}19^{\text{s}}$ earlier than a star with coordinates $\alpha = 0^{\text{h}}19^{\text{m}}4^{\text{s}}$, $\delta = +0^{\circ}13'2''$. The planet's zenith distance at that moment exceeded the star's zenith distance at its upper culmination by $19.4'$. What are the planet's coordinates?
89. What is the noonday altitude of the Sun in Moscow ($\varphi = 55^{\circ}45'$) on the day of the summer solstice?
90. At the mouth of the White Sea–Baltic Canal, the altitude of the celestial pole is $64^{\circ}33'$. What is the Sun's altitude at noon there on 22 December?
91. What is the Sun's meridian altitude on the day of the summer and winter solstice in:
 - a) Leningrad ($\varphi = 59^{\circ}57'$)
 - b) Tashkent ($\varphi = 41^{\circ}20'$)
 - c) Latitude $+23^{\circ}27'$
 - d) Latitude $+66^{\circ}33'$

What is remarkable about the last two circles of latitude, and what are their names?

92. How long does it take the Sun, due to its annual

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- motion along the ecliptic, to move a distance equal to its own diameter?
93. Name the constellation that is at the zenith in your locality at midnight on 8 March.
 94. Which constellation passes through the zenith at 10 p.m. on 23 September at your observing location?
 95. The right ascension of the first star is $5^{\text{h}}29^{\text{m}}$, that of the second is $10^{\text{h}}31^{\text{m}}$. Determine how much time after the first star the second star will culminate.
 96. If today a certain star culminated at $8^{\text{h}}08^{\text{m}}$, when will it culminate in 10 days' time?
 97. On what date does a vertically standing pole at the Tropic of Capricorn cast no shadow at noon?
 98. When do the Sun's right ascension and declination change most rapidly?
 99. Determine which constellations, in your location, rise, cross the southern meridian, and set at 6 p.m. on 22 December.
 100. Which of the circumpolar constellations in your locality cross the meridian (north and south) at about midnight on 7 November?
 101. Using a star chart with the ecliptic drawn on it, indicate approximately the point where the Sun is located on the day of the exercise, and the point where it will be a month later.

1.6 Time, Longitude and Calendar

102. In which direction do the stars move daily for an observer in northern latitudes facing the north celestial pole?
103. How far does the Sun's centre sink below the horizon at midnight on 22 June in Arkhangelsk ($\varphi = 64^{\circ}35'$)?
104. What are the Sun's right ascension and declination on 22 March and 23 September?
105. What are the right ascension and declination of the centre of the Sun on 22 June? What are they on 22 December?
106. Where on the celestial sphere is a star that describes, above the horizon, an apparent arc of 180° ? At which points of the horizon does it rise and set?

1.6 Time Measurement, Geographic Longitude, Calendar

107. Starting from which geographic parallel do the evening and morning astronomical twilights during the night not merge with each other?
Note: Astronomical twilight ends when the Sun descends 18° below the horizon. Civil twilight ends when the Sun descends 6° below the horizon.
108. From which geographic latitude do white nights

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- begin, i.e. civil twilight does not end (see note to problem 107)?
109. In which latitudes on 13 November does the Sun not rise if on that day its polar distance from the north celestial pole is 108° ?
 110. When during the day is the zenith distance of the Sun equal to 90° ?
 111. Can a star rise in the north-east and set
 - (a) in the north-west?
 - (b) in the south-west?
 112. If a star has risen at the north-east point of the horizon, at which point of the horizon will it set?
 113. At the moment when in Delhi the time is $16^{\text{h}}37^{\text{m}}$, in Madrid it is $11^{\text{h}}42^{\text{m}}9^{\text{s}}$ mean local time. Determine the difference in geographic longitudes between Delhi and Madrid.
 114. Determine the geographic longitude of the observation point if the upper culmination of the Sun on its meridian was observed on 23 November at $17^{\text{h}}56^{\text{m}}$ Moscow decree time.
 115. Travellers noticed that, by local time, a lunar eclipse began at $5^{\text{h}}13^{\text{m}}$, whereas according to the astronomical calendar this eclipse should have occurred at $3^{\text{h}}51^{\text{m}}$ Greenwich Mean Time. What is the longitude of their location?

1.6 Time, Longitude and Calendar

116. Determine the local time at a location if the time difference between that location and Moscow is $7^{\text{h}}46^{\text{m}}$, and the Moscow decree time is $18^{\text{h}}38^{\text{m}}$.
117. A clock set to decree time shows $23^{\text{h}}40^{\text{m}}$ at the moment of culmination of a star. What will this clock show after 10 days when observing the same event?
118. In Kharkiv it is noon, while in Kazan at the same moment the clocks show $12^{\text{h}}46^{\text{m}}$. What is the longitude of Kazan from Greenwich? (The longitude of Kharkiv from Greenwich is $2^{\text{h}}25^{\text{m}}$.)
119. In Kazan the clocks running on mean local time show $23^{\text{h}}46^{\text{m}}$. What should the clocks, running on mean local time in Novosibirsk, show at exactly the same moment?
120. According to calculations, an artificial Earth satellite passes over a given point of the celestial sphere in Kuibyshev (III time zone) at $17^{\text{h}}36^{\text{m}}$ Universal Time (i.e., Greenwich Mean Time). Determine the moment of its passage through this point in decree time.
121. On 22 December, at true noon, the captain of a ship measured the zenith distance of the Sun and found it equal to $66^{\circ}33'$. The chronometer, running on Greenwich Mean Time, showed $11^{\text{h}}54^{\text{m}}$ a.m. at the

1 Introduction

- moment of observation. Indicate on the terrestrial globe the location of the ship.
122. How many days did the year 1918 contain in the RSFSR in connection with the calendar reform?
 123. Why does the day of the vernal equinox not always fall on 21 March?
 124. What was the date in the new style when, in the old style, it was considered to be 15 February 1900? 25 February 1900? 5 March 1900?
 125. An aeroplane flies at a speed of 500 km/h from Magadan to Montreal, the distance between them being 800 km. When will it arrive in Montreal, if it leaves Magadan on 28 March at 23^h?

2 Structure of the Solar System

2.1 Laws of Planetary Motion. Configurations and Synodic Periods of the Planets

The solution of problems in this section is based on the use of:

1. The formula that determines the relationship between the sidereal period T and the synodic period S of a planet, and the Earth's orbital period T_E , expressed in years or days:
 - a) For an *outer planet*, the formula has the form:

$$\frac{1}{S} = \frac{1}{T_E} - \frac{1}{T},$$

2 Structure of the Solar System

b) For an *inner planet*, the formula has the form:

$$\frac{1}{S} = \frac{1}{T} - \frac{1}{T_E};$$

2. Kepler's Third Law, relating the sidereal period T of a planet's revolution to the semi-major axis a of its orbit:

$$\frac{T_1^2}{T_2^2} = \frac{a_1^3}{a_2^3}.$$

When solving problems, the unknown motion is compared with an already known one through the application of Kepler's laws and the formulas for the synodic period of revolution.

- 126. Can Mercury be visible in the evenings in the east?
- 127. A planet is seen at an angular distance of 120° from the Sun. Is this an upper planet or a lower planet?
- 128. On 19 May there was an opposition of Mars. In which constellation was it visible?
- 129. Venus was at greatest eastern (evening) elongation on 5 February. In which constellation was it located?
- 130. Is it possible to observe Venus in the morning in the west, and in the evening in the east? Explain the answer.

2.1 Planetary Motion and Periods

131. The angular distance of a planet from the Sun measured with a sextant turned out to be 55° . Determine whether this planet is an upper planet or a lower planet.
132. How great is the mean distance from the Sun of a minor planet whose period of revolution around the Sun is 8 years?
133. The asteroid Vesta completes a full revolution around the Sun in 3.63 years. By what factor, on average, is it farther from the Sun than the Earth?
134. What would be the synodic period of revolution of a planet whose sidereal period of revolution around the Sun would be 370 days? To what distance would it approach the Earth?
Hint. Take the Earth's sidereal period of revolution as 365 days; assume the orbits are circles.
135. An observer noted that a certain planet is in opposition every $665 \frac{1}{4}$ days. What is its distance from the Sun in astronomical units?
136. What would be the semi-major axis of a planet's orbit if its synodic period were equal to one year?
137. Do the familiar configurations of the constellations, known from observations on Earth, remain the same when observed from the surface of Mars?
138. Knowing that the period of revolution of Phobos is $7^{\text{h}} 40^{\text{m}}$, and that the axial rotation period of Mars

2 Structure of the Solar System

- is $24^{\text{h}} 37^{\text{m}}$, indicate how many times during a Martian day Phobos rises, and over which part of the Martian horizon it rises.
139. Determine the sidereal period of Mars, given that its synodic period is 780 days.
 140. Given the sidereal orbital periods of the Earth (365.3 days) and Mars (687.0 days) around the Sun, determine at what intervals of time the Earth would be seen from Mars as a morning star.
 141. The synodic period of an imaginary planet is 3 years. What is its sidereal period about the Sun?
 142. To the inhabitants of Mars (if any) or to cosmonauts there, the Sun's disk would appear under an angle of $21.7'$. Knowing that light from the Sun reaches the Earth in 8^{m} , and that the Sun's disk is seen from the Earth under an angle of $31.0'$, find how long light from the Sun takes to reach Mars.
 143. How long is required to travel from the Earth to Mars in an interplanetary craft moving on an orbit whose perihelion distance equals the Earth–Sun distance (1 AU) and whose aphelion distance equals the Mars–Sun distance (1.5 AU)?
 144. Can there occur a transit across the Sun's disk of Mars, of Mercury, or of Jupiter?
 145. It is known that Mars's satellites Phobos and Deimos revolve about it at mean distances of 9400 km and

2.1 Planetary Motion and Periods

23 600 km, respectively. Determine the orbital period of Deimos if the orbital period of Phobos is $7^{\text{h}} 40^{\text{m}}$.

146. What must be the lengths of the sidereal and synodic periods of a planet in the case where they are equal?
147. Compute the distance of Mars from the Earth at opposition and at conjunction, given that the mean distances of these planets from the Sun are 228 and 150 million km, respectively.
148. Determine the distance of Venus from the Sun in astronomical units if its greatest elongation from the Sun is 47° .
Hint. Assume the planetary orbits are circular. To solve the problem, draw a diagram showing the planetary orbits and Venus at greatest elongation.
149. Taking Mercury's orbit to be a circle, compute its mean distance from the Sun (in AU), given that at mean elongation Mercury recedes from the Sun by 23° .
150. For future cosmonauts on Mars, the Earth appears sometimes as a morning star and sometimes as an evening star, like Venus for the Earth. At what intervals of time can the Earth be seen from Mars culminating at midnight?
151. Knowing the distances of the planets from the

2 Structure of the Solar System

Sun, compute the greatest angular separation of the Earth from the Sun as seen from Mars.

152. By how large an angle can the Moon depart from the Earth for an observer located on Mars during mean opposition, if the distances from the Earth to the Moon and to Mars are taken as known?

2.2 Conditions of Visibility of the Planets

¹ The heliocentric longitude of a planet is the angle between the direction from the Sun to the point of the vernal equinox and the direction to the planet. This angle increases as the planet moves along its orbit.

153. Determine the nearest day of best morning visibility of Venus, given the heliocentric longitudes¹ on 1 January: Venus = $260^{\circ}40'$ and Earth = $99^{\circ}55'$.

154. The heliocentric longitude of the Earth on 1 January is 100° , and that of Mars is $172^{\circ}49'$. Determine the apparent position of Mars in the sky on 1 May of the same year and on 1 May two years later.

Hint. First determine the true (heliocentric) position of the Earth.

155. The opposition of Jupiter occurred on 15 July. When will the next opposition occur?

Hint. Use the equation of synodic motion.

156. An observer noted that a certain planet recedes eastwards from the Sun by 90° every 505 days. What are its orbital period about the Sun and the

2.3 Gravitation And Masses Of Celestial Bodies

radius of its orbit?

157. The best evening visibility of Venus (its greatest eastern elongation) occurred on 5 February. When will the next nearest such best visibility under the same conditions occur, if the sidereal period of Venus about the Sun is 225 days?

2.3 Gravitation And Masses Of Celestial Bodies

These problems concern the application of the law of universal gravitation and the exact form of Kepler's third law which includes the masses of the bodies.

The law of universal gravitation:

$$F = G \frac{m_1 m_2}{r^2}.$$

Kepler's third law in its exact form (including the masses M_1 and M_2):

$$\frac{T_1^2}{T_2^2} \frac{M_1 + m_1}{M_2 + m_2} = \frac{a_1^3}{a_2^3}.$$

This formula is convenient when comparing two systems in which the satellite masses are negligible compared with the mass of the primary body.

2 Structure of the Solar System

158. What is greater: the acceleration that the Earth imparts to the Sun, or the acceleration that the Earth imparts to the Moon, and by what factor?
159. The masses of the Earth and the Moon are in the ratio 81 : 1, and their radii are in the ratio 1 : 0.27. What is the acceleration of free fall on the Moon?
160. Calculate the mass of the Sun if the angular velocity of the Earth's motion is 1° per day. Use the gravitational constant $G = 6.67 \times 10^{-11} \text{ m}^3/(\text{kg s}^2)$ and the distance to the sun as $R = 1.49 \times 10^8 \text{ km}$
161. Determine the escape velocity (second cosmic velocity) from the surface of the Moon by comparing with the corresponding value for the Earth.
162. How long would it take Pluto, falling into the Sun, to reach it?
Hint. Assume fall along the major axis of an extremely elongated ellipse with the Sun at a focus, Pluto starting from aphelion. Take Pluto's distance from the Sun as 40 AU.
163. Calculate the mass of Neptune in units of the Earth mass, given that Neptune's satellite Triton orbits at a distance of 354 000 km from the planet's centre with a period of $5^{\text{d}} 21^{\text{h}}$.
Hint. Perform the calculation by comparing Triton's motion with the motion of the Moon about the Earth.

2.3 Gravitation And Masses Of Celestial Bodies

164. Determine the mass of Uranus in units of the Earth mass by comparing the motion of the Moon about the Earth with the motion of Uranus's satellite Titania, which orbits at a distance of 438 000 km with period $8^d 17^h$.
165. Compute the mass of Mars relative to the Earth from the motion of its satellite Phobos, for which $a = 9300$ km and $T = 0.32$ d. Take the corresponding lunar values as $a_{\text{Moon}} = 384\,000$ km and $T_{\text{Moon}} = 27.3$ d.
166. With what period would the Earth orbit the Sun at a distance of 1 AU if the Sun's mass were twice its present value?
167. By what factor would a person weigh less on Mars than on Earth if Mars's mass is 0.107 times the Earth's mass and its radius is approximately 3400 km?
168. The Moon at apogee is $\frac{1}{9}$ farther from the Earth than at perigee. By what percentage is the tidal force at perigee greater than at apogee?
169. By what factor should the tidal range at syzygy (new and full Moon) theoretically exceed the tidal range at the quarters of the Moon?
Note. Syzygies are new Moon and full Moon.
170. At what time of year is the Sun's tidal effect greatest, and at what time is it least? Compare these two values.

2 Structure of the Solar System

- Note.* The eccentricity of the Earth's orbit is $1/60$.
171. What speed must a body have in order to orbit the Sun just above its surface?
- Hint.* Use Kepler's third law and treat the required speed as the limiting (orbital) speed at which the body would no longer fall onto the Sun's surface. Use the Sun's mass and radius from the textbook tables.
172. What must be the orbital period of a body moving in a circular orbit just above the Sun's surface?
- Hint.* Use Kepler's third law. Take the necessary solar data from the textbook tables.
173. If the Earth had a satellite with an orbital period equal to eight lunar months, what would be its distance from the Earth?

2.4 Elements of Astronautics

174. Could the orbital period of an artificial Earth satellite, moving in accordance with Kepler's laws, be equal to 81 minutes?
175. Why are objects weightless inside a spacecraft flying with its engines switched off?
176. Can objects be weighed on balance or spring scales inside a spacecraft orbiting the Earth in a circular

2.4 Elements of Astronautics

path at an altitude of 100 km? And at an altitude of 1000 km?

177. On 9 January 1978, the experimental satellite *Cosmos-979*, launched on 27 December 1977 with a perigee altitude of 210 km and an apogee altitude of 348 km, ceased to exist. What was its orbital period?
178. Cosmonaut V.V. Tereshkova flew over Berlin in a spacecraft at an altitude of 170 km. Could she have seen Moscow at the same time, given that it is 2000 km from Berlin?
- Hint.* To determine the “horizon distance”, draw a diagram with tangents from the spacecraft to the Earth’s surface.
179. How many orbits must a spacecraft on a polar circular orbit at an altitude of 271 km make before landing back in its launch area?
- Note.* An Earth satellite whose orbital plane passes over the Earth’s poles is called a *polar satellite*.
180. What is the speed of artificial Earth satellites in circular orbits at altitudes of 125, 250, 500, and 1000 km? Calculate the same for artificial satellites of Mars.
181. If the altitude of a polar satellite is 250 km (see note to problem 179), by how many kilometres to the west will the projection of its orbit onto the Earth’s surface be shifted per revolution at a

2 Structure of the Solar System

latitude of $57^{\circ}37'$?

182. The spacecraft *Soyuz-26*, launched on 10 December 1977 and docked with the space station *Salyut-6* on 11 December, landed on 16 January 1978. Its perigee altitude was 267 km, and its orbital period was 90.2 minutes. What was its apogee altitude?
183. Why did the first three Soviet artificial Earth satellites fail to become permanent satellites like the Moon, and instead exist for only a relatively short time?
184. What would be the speed of an artificial Moon satellite orbiting 50 km above its surface?
185. After separation of the launch vehicle from the satellite, the launch vehicle at first moves behind the satellite and then overtakes it. Why?
186. The Soviet interplanetary stations “Mars-2” and “Mars-3”, launched towards Mars in May 1971, reached it and became artificial satellites of Mars by command from Earth after 192 and 188 days of flight, respectively. By how much was their flight longer than it would have been along a half-elliptical orbit corresponding to the average distance of Mars from Earth at opposition?
187. For the Soviet interplanetary stations “Mars-2” and “Mars-3”, the minimum distances from the surface of Mars and the orbital periods were, re-

2.5 Earth

spectively, 1380 km and 18 h, and 1500 km and 1 day. What was the maximum distance of the satellites from the surface of Mars?

188. The opposition of Mars occurred on 10 August 1971. The Soviet interplanetary stations “Mars-2” and “Mars-3” departed towards it on 27 November and 2 December 1971, respectively. Draw a diagram showing the relative positions of the Sun, Earth, and Mars at that time, assuming circular planetary orbits.
189. Calculate the linear velocities of a spacecraft in perigee and in apogee if it is known that in perigee it passes at an altitude of 227 km above the ocean surface and the major axis of its orbit is 13 900 km.

2.5 Earth

190. If the Earth’s daily rotation gradually slows down over centuries (as has recently been established), will astronomical phenomena measured by clocks regulated by Earth’s rotation occur faster or slower than before?
191. If it were not the Earth but the sky that rotated, what speed would the star α Centauri need to have in its daily motion around the Earth, given that

2 Structure of the Solar System

- its light reaches us in 4 years and its declination is -60° ?
192. At what latitude will the eastward deflection of a body falling from a given height be greatest?
 193. Calculate the eastward deflection of a body falling from a tower 100 m high built on the equator.
 194. Calculate the displacement of a Foucault pendulum in 1 hour at latitudes 30° , 60° , 45° , 75° , 89° , 90° , 0° , and in your city.
 195. What is the speed of Leningrad ($\varphi = 59^\circ 57'$) due to the Earth's daily rotation? (Mean radius of the Earth $R = 6371$ km.)
 196. What are the speeds of points on the Earth's surface due to its daily rotation:
 - (a) on the equator;
 - (b) at latitude $\varphi = 48^\circ 24'$, if the Earth's mean radius is 6371 km?
 197. At what latitude is a place located that moves, due to Earth's daily rotation, at a speed half that of Moscow ($\varphi \approx 56^\circ$)? At twice the speed of Murmansk ($\varphi = 69^\circ$)?
 198. How would the seasons qualitatively change if the eccentricity of Earth's orbit increased to 0.5?
 199. If the Earth's axis of rotation were perpendicular to the plane of the ecliptic, how would the length

2.5 Earth

of the day vary at different locations throughout the year? How would this affect the seasons and the climates of the Earth?

200. At what tilt of the Earth's axis to the plane of its orbit would the Antarctic Circle coincide with the Tropic of Capricorn?
201. How great is the Earth's speed in its motion around the Sun, assuming this motion is uniform along a circle of radius 149 600 000 km, with an orbital period of 365 $\frac{1}{4}$ days?
202. By how much would the length of the year change if the Earth's distance from the Sun increased by 1 m? Assume the Earth's orbital speed remains unchanged.
203. If the Earth were represented as a globe with a diameter of 3 m, how would its oblateness be expressed in that scale?
204. Newton's proof of the Earth's oblateness assumes beforehand that the Earth rotates. Which proofs do not depend on this assumption?
205. How great is the centripetal acceleration of a body placed on the Earth's equator at sea level? Given: Earth's radius $R = 6378$ km, rotation period $T = 86\,164$ mean solar seconds.
206. How great is the acceleration due to gravity at the equator if a seconds pendulum there has a length

2 Structure of the Solar System

$$l = 991.03 \text{ mm?}$$

207. At the equator, the acceleration due to gravity is $g_0 = 9.781 \text{ m/s}^2$; at latitude φ it is

$$g_\varphi = 9.781 (1 + 0.00512 \sin^2 \varphi) \text{ m/s}^2.$$

Calculate the length of a seconds pendulum in Dnipro ($\varphi = 48^\circ 20'$).

208. If the Earth's surface were devoid of water, how would this affect the mean temperature, the diurnal temperature variation at a given location, and the temperature variation over the entire Earth's surface?
209. If the mass of the Earth's atmosphere were reduced, how would this affect the mean temperature, the diurnal temperature variation at a given location, and the temperature variation over the entire Earth's surface?
210. Compare the brightness of illumination of equal areas on the equator, the Tropic of Cancer, and the Arctic Circle during the equinoxes at noon, when the Sun reaches its greatest altitude above the horizon at the respective locations.
211. What should be (theoretically) the ratio of the intensities of illumination and heating of horizontal ground at noon in Saratov ($\varphi = 51^\circ 32'$) on 22 June and 23 September?

2.5 Earth

212. Compare the amounts of heat received by the ground on the days of the summer and winter solstices for locations at latitudes 65° , 45° , 23.5° , and 0° . Compare the ratios obtained for each location with one another and relate them to known geographical data.
213. Compare the ratios of the heat received per unit surface area in Moscow ($\varphi = 55^\circ 45'$) on the days of the summer and winter solstices, and on the days when the Earth passes through perihelion (in winter) and aphelion (in summer). By how many times is the influence of the inclination of solar rays greater than the influence of the change in distance from the Sun (in Moscow)?
- Note:* Eccentricity of Earth's orbit $e = 0.017$.
214. At the equator the acceleration due to gravity is 9.781 m/s^2 . By what factor would the Earth have to rotate faster for bodies at the equator to be weightless?
215. Calculate the linear distance between points A and B along a parallel of the Earth, if the geographical coordinates of the points are respectively

$$A : \varphi = 45^\circ \text{ N}, \lambda = 35^\circ \text{ E},$$

$$B : \varphi = 45^\circ \text{ N}, \lambda = 70^\circ \text{ W}.$$

2.6 Determination of Distances to Bodies of the Solar System and Their Sizes. Parallax

When solving these problems, remember that the *horizontal parallax* is the angle under which the radius of the Earth, taken perpendicular to the line of sight, is seen from the celestial body. The parallax varies inversely with the distance D to the body:

$$D = \frac{R_E}{\sin p},$$

where R_E is the Earth's radius and p the horizontal parallax. Knowing the distance D to a body and measuring its angular diameter ρ (provided the body appears as a disc), one may compute its linear diameter d by the exact relation

$$r = D \sin \rho,$$

and for small angles

$$r = \frac{\rho}{p} R_3.$$

216. What is the angular diameter of the Sun as seen from Neptune? By what factor is solar illumination on Neptune weaker than on the Earth?

2.6 Distances and Parallax

217. What is the distance from the observer to Venus if its angular diameter equals $1'$ and its linear diameter is 12 200 km?
218. To what angular separation from the Sun can the Earth recede as seen by an observer on Mars? On Neptune?
219. The diameter of the Moon is 0.27 of the Earth's diameter. Neglecting the Earth–Moon distance, determine the horizontal parallax of the Sun for an observer standing on the Moon.
220. What is the horizontal parallax of Mars when that planet is at its nearest approach to the Earth 0.378 AU? The horizontal parallax of the Sun is $8.8''$.
221. What is the largest angular diameter of the Earth as seen from Mars when the Earth is at a distance of 0.378 au?
222. Neptune is at a distance of 30 au from the Sun. What is its horizontal parallax at the mean distance from the Earth? What is Neptune's annual parallax?
223. What is the horizontal parallax of Jupiter when it is at a distance of 6 au from the Earth?
224. At the moment of opposition Jupiter was 628×10^6 km from the Earth; its angular diameter at that time was $47.2''$. From these data determine Jupiter's linear radius.

2 Structure of the Solar System

- 225. The least distance of Venus from the Earth is 40×10^6 km; at that moment the angular radius of Venus is $32.4''$. From this determine the linear diameter of that planet.
- 226. Given that the horizontal diurnal parallax of the Moon is $57^\circ 2.7'$ and the angular radius of the Moon is $15^\circ 32.6'$, compute the distance to the Moon and its linear radius expressed in Earth radii, and also give the surface area and volume of the Moon compared with those of the Earth.
- 227. The equatorial horizontal parallax of the Sun is $8.79''$ with an uncertainty of $0.01''$. State, in percent and in kilometres, the accuracy with which the distance to the Sun is determined from these data.
- 228. The parallax of the Sun is $8.8''$, and its apparent radius is $16' 1''$. By what factor is the radius of the Sun larger than the radius of the Earth? What is the diameter of the Sun in kilometres?
- 229. What is the angular diameter of the Earth as seen from Jupiter during an Earth transit of the solar disc? (Given that Jupiter's distance from the Sun is 5.2 au and the Sun's parallax is $8.8''$.)
- 230. Under what angle is Venus seen from the Earth during superior conjunction and during transit across the Sun's disc? Under what angle is the

2.6 Distances and Parallax

Earth seen from Venus at those times? Under what angle is the Moon seen from Venus? (Use Sun parallax $8.8''$; Venus–Sun distance 0.72 au; lunar diameter = 0.27 Earth diameters; Venus diameter = 0.95 Earth diameters.)

231. Prove that at mean opposition Mars, as seen from the Earth, is seen under almost the same angular diameter as the Moon as seen from Venus. Why, under these conditions, should the Moon from Venus appear brighter than Mars from the Earth?
232. On one occasion the apparent radius of Jupiter was measured with a micrometer as $17.75''$. The computed distance to the planet that day (its orbital elements are known) was 5.431 au. Determine Jupiter's diameter relative to the Earth's, given Sun parallax $8.80''$.
233. At the closest approach of Mars to the Earth (56×10^6 km) its angular diameter is $25''$. What is its linear diameter?

2 Structure of the Solar System

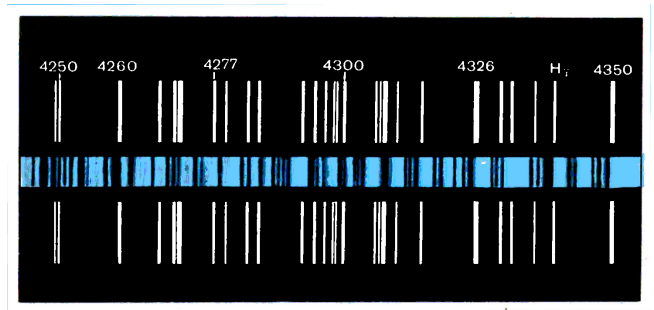


Figure 9: Displacement of the H line in the spectrum of a star moving in the direction of the line of sight. Above and below are shown the comparison spectra obtained in the laboratory. Above these are indicated the wavelengths expressed in angstroms ($1 \text{ \AA} = 0.0001 \mu\text{m}$).

3 Physical Nature of the Bodies of the Solar System

3.1 Spectral Analysis

When solving problems in this section you should use the Doppler principle:

$$\lambda = \lambda_0 \left(1 + \frac{v}{c} \right),$$

where v is the radial velocity of the source relative to the observer (take v positive for recession and negative for approach), λ_0 is the laboratory wavelength and λ the observed wavelength.

When working with spectrograms remember that the *dispersion* of the spectrum is the number of Å contained in 1 mm on the spectrogram (i.e. Å/mm).

234. Using the spectrogram shown in Figure 41 ([Figure 9](#) here) of the textbook, estimate the dispersion

3 *Physical Nature of the Bodies of the Solar System*

for each region of the spectrum. Measure the displacements of the iron absorption lines in the stellar spectrum relative to the corresponding lines in the comparison spectrum with an accuracy of ± 1 mm.

Instruction: Measure the shifts of the centres of the dark lines in the star's spectrum relative to the mid-points of the bright lines in the comparison spectrum. Multiply these measured displacements by the number of $\text{\AA}$ per mm in the given region of the spectrogram to obtain the difference of wavelengths (stellar minus laboratory) in $\text{\AA}$. From these differences determine the radial velocity of the star with respect to the Earth.

Hint: Use several lines and adopt the mean of the obtained velocities as the final result.

3.1 Spectral Analysis

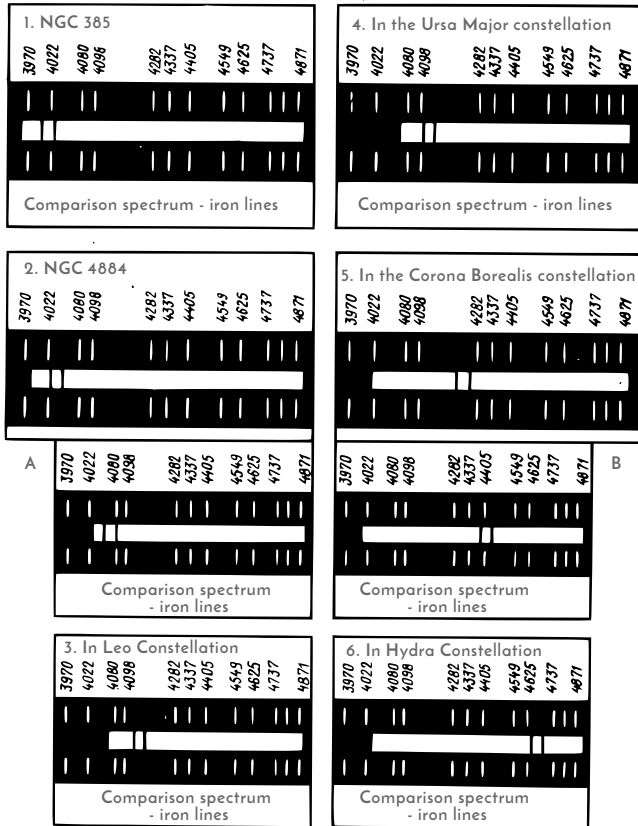


Figure 10: Redshifts of lines in the spectra of distant stellar systems – galaxies. From the data of these spectra, determine the velocities of the galaxies, using the bright emission lines in the comparison spectra.

3 Physical Nature of the Bodies of the Solar System

235. On a series of photographs of the spectra of distant stellar systems (galaxies) (see [Figure 10](#)), measure the large shifts of the H and K absorption lines of ionised calcium relative to the bright iron lines in the terrestrial comparison spectrum, and from them determine the velocities of the galaxies.
- Instruction:* The laboratory wavelengths of the H and K lines under terrestrial conditions are $3968 \text{ \AA}$ and $3964 \text{ \AA}$ respectively.
- Estimate the spectral scale (the number of $\text{\AA}$ per mm on the photograph) by measuring with a ruler (in mm) the distance between the lines in the comparison spectrum nearest to H and K and located on both sides of H and K . Determine the wavelengths of the shifted H and K lines by graphical interpolation on a scaled drawing of that portion of the spectrum made on graph paper.
236. The dispersion of a star's spectrogram is $60 \text{ \AA/mm}$ near H_γ . That line in the star's spectrum is shifted by 0.005 mm towards the violet relative to its normal (laboratory) position. What is the star's velocity in km/s according to the Doppler principle? The laboratory wavelength of the H_γ line is $4341 \text{ \AA}$.
237. Suppose the spectrograph dispersion is linear and the whole visible stellar spectrum (from violet to red) occupies a length of 5 cm on the plate. What

3.1 Spectral Analysis

linear displacement on the plate will the spectral line with laboratory wavelength $\lambda_0 = 5000 \text{ \AA}$ have if the star is approaching us with a speed of 10 km/s ?

238. In the spectrum of a star the calcium line ($\lambda_0 = 4227 \text{ \AA}$) is shifted by $0.70 \text{ \AA}$ towards the violet. Determine the star's velocity and state whether it is approaching or receding.
239. The *K* line of ionised calcium in the violet part of the spectrum (laboratory wavelength $\lambda_0 = 3934 \text{ \AA}$) is shifted by $4 \text{ \AA}$ to the blue in the star's spectrum. In which direction and with what speed is the star moving?
240. In the spectrum of a comet's head the yellow line of sodium vapour with laboratory wavelength $\lambda_0 = 5893 \text{ \AA}$ is shifted by $0.6 \text{ \AA}$ to the red. Compute the comet's speed using the Doppler formula.
241. If a star lies in the plane of the ecliptic and the Earth moves on its orbit directly towards that star, and six months later moves directly away from it, by how much will the wavelength of the helium line $\lambda_0 = 5876 \text{ \AA}$ change over the six months of observation?
242. Is it possible, using the Doppler principle, to measure the launch speed of a space rocket by means of its radio transmitter?

3 *Physical Nature of the Bodies of the Solar System*

243. With what accuracy (in mm) must the position of a line in the spectrum be measured in order to determine the star's velocity from the sodium line ($\lambda_0 = 5893 \text{ \AA}$) to an accuracy of 1 km/s, if the spectrograph dispersion is $300 \text{ \AA/mm}$?

3.2 Planets

Here are questions based on the material already covered, which supplement the understanding of planetary physics.

244. Assuming the human eye can still resolve details subtending an angle of $0^\circ 2'$, calculate (in km) the size of the smallest details resolvable on Mars with a telescope of magnification $600\times$ during Martian opposition when its angular diameter is $25''$.
Note: It is worth noting that magnifications greater than $600\times$ are generally impractical for observing Mars because of atmospheric seeing.
245. How would the surface of a planet be divided into climatic zones if its rotation axis made an angle of approximately 65° with the plane of its orbit (as for Mars)?
246. Calculate the escape velocity required for a spacecraft to leave Mars.

3.2 Planets

247. What time interval (expressed in Martian solar days) elapses between two successive upper culminations of Phobos as observed from the surface of Mars? Given: Martian solar day = $24^{\text{h}} 37^{\text{m}}$ (mean solar time). Sidereal orbital period of Phobos = $7^{\text{h}} 39^{\text{m}}$ (mean solar time).
248. Suppose the entire mass of Saturn's rings were gathered into a single large satellite orbiting at the mean radius of the rings and the satellite had the same material density as the ring particles. Would the night illumination on Saturn, in places where the rings are well seen, become stronger or weaker?
249. Why does the terminator of Venus², when observed from Earth, appear in the shape of an elliptical arc?
250. In which configurations does the disc of a superior planet exhibit the greatest phase defect?
251. Assuming that Uranus's rotation axis lies in the plane of its orbit, describe the alternation of day and night, as well as the change of seasons on this planet, given that its rotation period is 10.8 h and its orbital period around the Sun is 84 Earth years.
252. Suppose that Earth and Neptune are approximately aligned between the Sun and the nearest star. By what factor would the star appear brighter when viewed from Neptune than from Earth?
253. What is the synodic period of Deimos for an ob-

² The terminator of Venus is the boundary line on Venus that separates the sunlit half of the planet from the half in darkness – essentially, the line between day and night on Venus.

3 Physical Nature of the Bodies of the Solar System

- server located on Mars?
254. Phobos, the inner satellite of Mars, is at a distance of 2.8 planetary radii from the planet's centre. The diameter of Mars is 2.0 times the diameter of the Moon. By what factor does the angular diameter of Mars, as seen from Phobos, exceed the apparent diameter of the Moon as seen from Earth?
255. What is the appearance of Saturn's rings for observers situated at Saturn's equator and at its poles?

3.3 Comets and Meteors

256. How can one distinguish, in observation, a comet without a tail from an ordinary nebula?
257. Which observations would prove that comets are not located in the Earth's atmosphere, as was assumed in antiquity?
258. Is it possible to determine the length of a comet's tail in kilometres with high precision?
259. Using the drawing of Donati's comet of 1858 ([Figure 11](#)), determine the length of the comet's tail in kilometres, given that the distance from the comet to the Earth is 57×10^6 km.
- Hint:* Assume the tail lies in a plane perpendicular to the line of sight.

3.3 Comets and Meteors

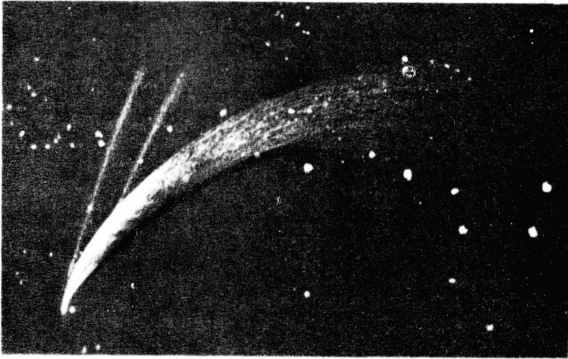


Figure 11: Donati's comet of 1858.

260. Can a comet that periodically returns to the Sun keep its appearance unchanged forever?
261. Using Kepler's Third Law, calculate the period of a comet whose aphelion is located at a distance of 140 000 au, which is about half the distance to the nearest known star.
262. The semi-major axis of Encke's comet is 2.22 au, and its eccentricity is 0.847. Draw the orbit of this comet and determine from the drawing its distances from the Sun at perihelion and at aphelion.
263. As a comet approached the Earth and the Sun, its brightness decreased. How can this phenomenon

3 Physical Nature of the Bodies of the Solar System

be explained?

264. One of the comets discovered by G.N. Neujmin passed perihelion on 9 March 1916. Later, comets with similar orbital elements were observed passing perihelion on 16 January 1927, 2 June 1932, and 1 May 1943. If these are different appearances of the same comet, what is its orbital period around the Sun?
265. When a bright meteor is observed flying among the stars, outshining them with its brilliance, how would this appear to an observer viewing the Earth from the Moon?
266. Meteors from the Leonid stream have a radiant at right ascension 10° and are observed on 14 November. At what time of night are they visible?
267. Knowing that the orbital period of the Leonid stream is $34 \frac{1}{4}$ year, determine the semi-major axis of their orbit.
268. Estimate the approximate width of the Perseid meteor stream in kilometres, given that it is observed from 16 July to 22 August.
269. Photographic observations of meteors show that their radiant occupies some area in the sky, meaning that their paths extended backwards do not strictly intersect at one point. What does this indicate?

3.4 The Moon, Its Motion and Nature

270. Prove that the true path length L of a meteor in the atmosphere is given by:

$$L = r_1^2 + r_2^2 - 2r_1r_2 \cos l,$$

where r_1 and r_2 are the distances from the observer to the start and end points of the meteor's path, and l is its apparent angular length.

271. A bolide, observed at a distance of 0.5 km from the observer, had an apparent disk half the angular size of the Moon. What was its actual diameter?
272. If the geographic locations of known meteorite falls in the Asian part of the USSR are plotted on a map, it turns out that almost all are located near the route of the Siberian Railway. How can this be explained?

3.4 The Moon, Its Motion and Nature

273. How many rotations around its own axis does the Moon make relative to the Sun in one year?
274. What is the length of a sidereal day on the Moon? How long do day and night last there?
275. The Moon takes at least 2 min to rise when observed from Earth. How long does the Earth take to rise for an observer on the Moon?

3 *Physical Nature of the Bodies of the Solar System*

276. Knowing the Moon's radius, determine the distance to the visible horizon on the lunar surface from a height of 1.7 m.
277. At the edge of the lunar disk, a mountain is seen protruding by $0.03'$. Given that the linear diameter of the Moon is 3476 km and its angular diameter is $30'$, find the height of this mountain in kilometres.
278. Describe how the “daily” and “annual” motions of the Earth and the Sun appear in the sky to an observer on the Moon.
279. Is it possible to see the Earth from the Moon during “new Earth” (when the Moon is in new moon phase as seen from Earth)?
280. The Earth has an albedo six times greater than that of the Moon. By how many times is Earthlight on the Moon brighter than moonlight on Earth? *Note:* The Moon's diameter is 0.272 that of Earth.
281. By how many times is the Moon's light at full moon weaker than sunlight, if the apparent magnitude of the Moon is -12.5 and that of the Sun is -26.7 ?
282. Give one reason why the Moon's light at first or last quarter is less than half of its illumination at full Moon.
283. Does the Sun appear the same when observed from the Moon as from the Earth?

3.4 *The Moon, Its Motion and Nature*

284. When many meteors are observed on Earth on 11–12 August, can they also be seen from the Moon?
285. Is it possible to observe lunar polar aurorae while on the Moon?
286. By what factor is the surface gravity on the Moon less than on the Earth, if the Moon's mass is $\frac{1}{81}$ of the Earth's and its radius is 3.7 times smaller?
287. Calculate the duration of the sidereal month, given that the Moon's mean orbital motion is $13^{\circ}11'$ per day.
288. Determine the Moon's mean orbital speed, assuming its orbit is circular with mean Earth–Moon distance 384 000 km and that the sidereal month contains 27.3 d.
289. Can the Moon be observed to rise on every calendar day?
290. In which seasons does the full Moon culminate at its highest and lowest altitudes at a given location?
291. Calculate the maximum possible altitude of the Moon above the horizon in Odessa and in Perm, given that the inclination of the lunar orbit to the ecliptic is $5^{\circ}9'$.
292. In which part of the sky is the full Moon located when observed 4 hour before midnight?
293. In what phase will the Moon be on the day of the autumnal equinox, if at that time its right ascen-

3 Physical Nature of the Bodies of the Solar System

sion is $18^{\text{h}}20^{\text{m}}$?

- 294. At what time of day is the Moon observed one week after new Moon?
- 295. On which limb of the Moon should one expect the occultation of a star by the Moon, and on which limb will the star reappear?
- 296. The Sea of Moscow, located on the far side of the Moon, has a diameter of about 300 km. Would it be visible to the naked eye from Earth if it were on the near side of the Moon, given that the resolving power of the human eye is $1'$?
- 297. Where in the sky does an astronaut, located at the centre of the hemisphere of the Moon visible from Earth, see the Earth?
- 298. Could an astronaut who has landed on the Moon navigate using a magnetic compass?
- 299. How does the starry sky appear to an astronaut standing on the Moon, and how does it appear from a spacecraft orbiting the Earth?

3.5 Eclipses

- 300. Which edge of the Sun's disk, eastern or western, first comes into contact with the Moon during a solar eclipse?

3.5 Eclipses

301. Can an annular solar eclipse occur when the Moon is at perigee? At apogee?
302. What does an astronaut on the Moon see during a solar eclipse?
303. When an observer on the Moon sees a partial eclipse of the Sun by the Earth, what does a terrestrial observer see at the same time?
304. Why can there be no annular lunar eclipse?
305. What features distinguish the partial phases of a lunar eclipse from the Moon's ordinary phases?
306. Can a total lunar eclipse be observed at local noon?
307. There are cases when the Moon, being in total eclipse, rises before the Sun sets, so that both bodies are visible simultaneously; therefore, the Sun should be visible from the Moon, and yet the Moon is in eclipse. How can this be explained?
308. Can the middle of a total lunar eclipse be observed at 4 a.m. local time?
309. If in the current year a lunar eclipse occurred in August, can another lunar eclipse occur in July of the following year? Can it occur in October of the current year, and if not, why?
310. Can a solar eclipse be observed in December from Murmansk ($\varphi = 69^\circ$)?
311. Why, during the partial phases of a solar eclipse, do the spots of light in the shade of foliage take on

3 *Physical Nature of the Bodies of the Solar System*

the shape of crescents?

312. Which type of eclipses, solar or lunar, are more often observed in Kyiv? In Irkutsk?
313. It has been calculated that the semi-major axis of the Moon's orbit is gradually increasing. When it increases by 10%, will total solar eclipses still occur on Earth?
314. Can Jupiter be occulted by the Moon during a lunar eclipse? What about Venus?
315. In 1963, two solar eclipses were observed: an annular one on 25 January and a total one on 20 July. Calculate the approximate dates of the nearest solar eclipses corresponding to these in the new saros cycle³.
316. Under what conditions will a central total lunar eclipse have the maximum possible duration?
317. In 1941 there were two solar and two lunar eclipses, namely: 13 March – partial lunar, 27 March – annular solar, 5 September – partial lunar, and 21 September – total solar eclipse. Using the saros cycle, predict when these eclipses occurred after 1941 and when they will occur again in the near future.
318. Using a map of the total solar eclipse path, determine the time of maximum eclipse and the magnitude of its greatest phase in Moscow. Do the same

³ A saros cycle is about 18 years, 11 days, and 8 hours, after which the Sun, Earth, and Moon return to nearly the same relative positions, producing a similar eclipse. A new saros eclipse is the next occurrence in the same cycle, shifted to a different part of the Earth by the extra 8 hours.

3.5 Eclipses

for Gorky, Omsk, and Tuapse.

Note: Such maps are included in several issues of the “School Astronomical Calendar”.

3 Physical Nature of the Bodies of the Solar System



Figure 12: Top: Mars (visible through a telescope). Bottom: The polar caps of Mars.

3.6 Exercises for Independent Study: Planets and Comets

Instruction: In problems where you need to know the radius of a planet or the Sun to scale (in mm), and the figure shows only part of the limb, determine the radius of curvature by trial: superpose the limb segment on tracing paper and find the circle which best fits the arc.

319. Using the coloured drawings of the polar cap of Mars made at monthly intervals (see the plates on the front endpapers of the textbook, [Figure 12](#)), determine the visible area of the polar cap. The diameter of Mars is 6800 km.

Hint. Neglect the facts that the cap is not entirely visible and that we see it in projection.

320. From the sketches of the Martian polar cap ([Figure 13](#)) construct a graph of its size variation during melting. The diameter of Mars is 6800 km.
321. Two drawings of Mars ([Figure 14](#)) taken through a telescope were made two hours apart. Identify common surface details and, by estimating the planet's angle of rotation during these two hours, approximately determine the length of the Martian solar day. *Hints:*

- (a) The rotation angle θ of a surface detail may be

3 Physical Nature of the Bodies of the Solar System

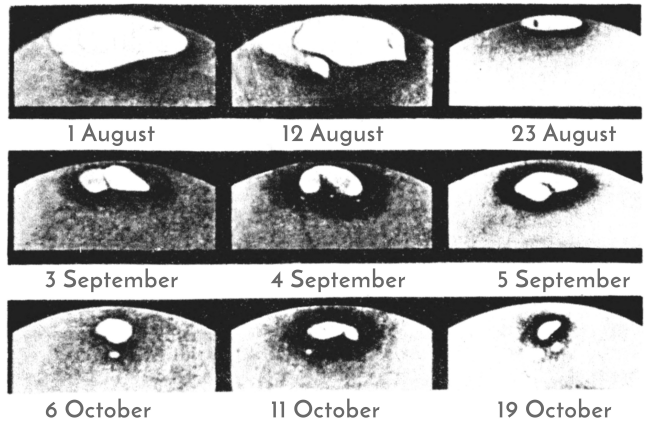


Figure 13: The polar caps of Mars observed at different times.

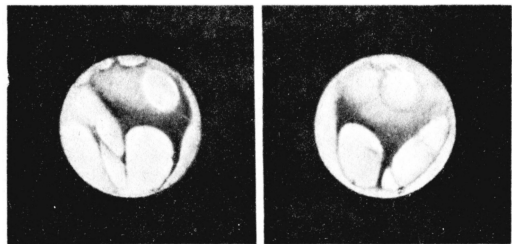


Figure 14: Sketches of observations Mars at different times.

3.6 Exercises for Independent Study: Planets and Comets

estimated from its apparent linear displacement along the chord measured from the central meridian; the displacement is $r \sin \theta$, where r is the radius of the circle described by that detail.

- (b) Remember that the observable disk of the planet is a projection of its surface onto the picture plane.

322.

- (a) By measuring positions of the same Jovian cloud details on two upper or on two lower drawings (see [Figure 15](#)), determine Jupiter's rotation period.
- (b) Do the same using [Figure 16](#).
- (c) Attempt to estimate Jupiter's rotation period from two sketches of its belts (see the central part of [Figure 15](#)), knowing that the period is much shorter than an Earth day.

Notes. (1) Apply the hints given for problem 3. (2) Answers for problems 3 and 4 are listed in Table V (Solar System) in the textbook. Your answer may be approximate; its accuracy improves with the precision of your measurements.

323. From the coloured drawing of Saturn (see the plate on the front endpapers of the textbook ([Figure 17](#))),

3 Physical Nature of the Bodies of the Solar System

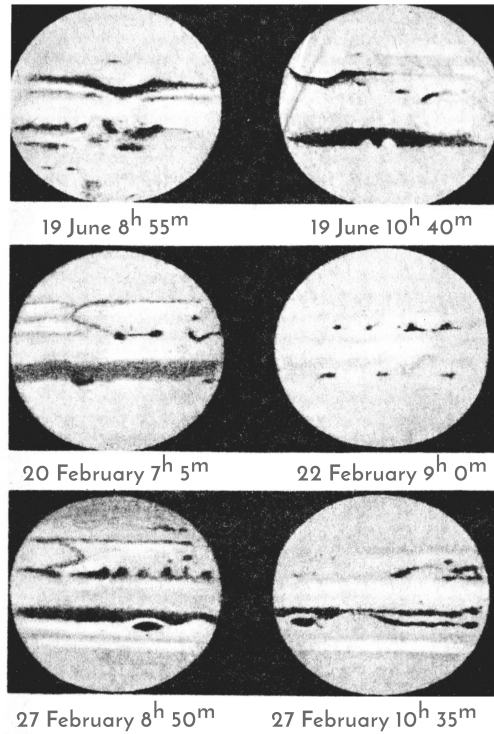


Figure 15: Three paired sketches of Jupiter's rotation.

3.6 Exercises for Independent Study: Planets and Comets

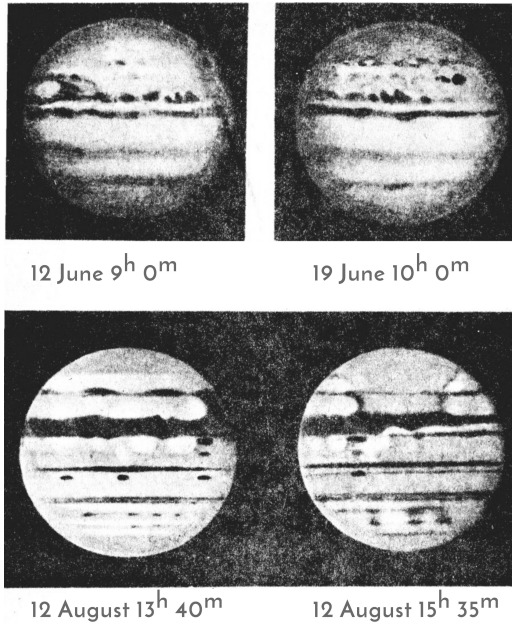


Figure 16: Two paired sketches of Jupiter's rotation.



Figure 17: Two paired sketches of Jupiter's rotation.

estimate (in km) the distance from Saturn's cloud tops to the inner edge of the rings and the widths of the individual rings, given Saturn's diameter is about 120 000 km.

324. (a) Assuming that [Figure 18](#) are 10× enlargements of photographs taken with a camera of focal length $f = 0.10$ m, estimate the angular *length* and *width* of the comet's tail (express the results in degrees).

3.6 Exercises for Independent Study: Planets and Comets

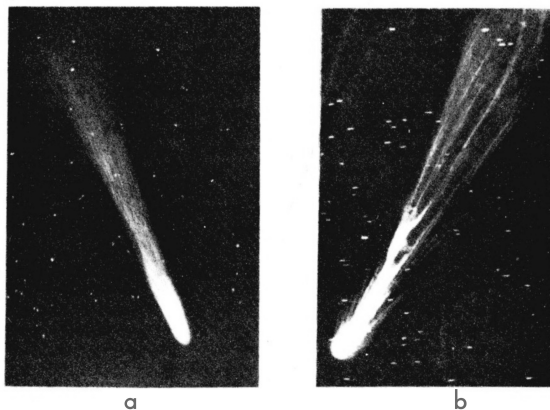


Figure 18: Photographs of two comets.

Instruction: First make a drawing of the path of the rays from the head and the tail of the comet in the sky towards the objective, and of their path from the objective to the image on the photographic plate.

- (b) Repeat the procedure for [Figure 11](#). Determine the plate scale by using the apparent size of the constellation Ursa Major visible on the same plate together with a star chart (coordinate grid). Convert the measured angular

3 Physical Nature of the Bodies of the Solar System

sizes to linear sizes in kilometres, assuming a horizontal parallax $10''$ for each comet.

4 The Sun and Stars

4.1 The Sun

Typical Problem: Is it possible to see on the Sun, with the unaided eye, a spot whose diameter is twice that of the Earth (naturally, through darkened or smoked glass)? And what about through a telescope with $20\times$ magnification? Take the resolving power of the eye as $3'$, and remember that the Sun's diameter is 100 times greater than that of the Earth.

Solution: One must always remember that the apparent angular diameter of the Sun (and the Moon), as seen from Earth, is about $30'$. Then a disc with 50 times smaller diameter at the same distance will have an angular diameter of about $0.6'$, i.e. 5 times smaller than the resolving power of the unaided eye. Therefore, the given spot cannot be seen without optical aid. In a telescope, however, it can be clearly seen.

325. Why in a certain year in the USSR were far more

4 The Sun and Stars

- auroras registered than four years earlier?
326. In 1957 a maximum of sunspots was observed. Indicate approximately the years of the nearest maximum and minimum of solar activity.
327. What angular diameter must a sunspot have in order that its linear diameter equals the radius of the Earth (6371 km)?
328. If the smallest sunspot visible to us in a telescope has a diameter of $0.7''$, what is its linear diameter?
329. How much meteoritic matter must fall daily on 1 m^2 of the Earth's surface with a speed of 40 km/s so that the heat imparted is equivalent to the solar heating, which on average equals $2.1 \times 10^4 \text{ J}/(\text{min m}^2)$? And how much must fall daily on 10 m^2 ?
330. The solar constant is $2 \text{ cal}/(\text{cm}^2 \text{ min})$, while on average the Earth receives 5 kcal per 1 m^2 per 1 min from the Sun. How many kilograms of meteoritic matter would need to fall yearly with a speed of 40 km/s on each square metre of the Earth's surface to generate an amount of heat equal to that received from the Sun by 1 m^2 in one year?
331. What mass, falling from a height of 100 m , will acquire at the end of its fall a kinetic energy equal to the energy of solar radiation received in 1 h by 10 m^2 of the Earth's surface, if on average the Earth receives from the Sun $2.1 \times 10^4 \text{ J}$ per 1 min per

4.1 The Sun

1 m^2 ?

332. For how long would vertically incident solar rays be able to melt an ice layer of thickness 1 cm covering the Earth, provided that solar heating of each 1 m^2 of the Earth's surface amounts to $2.1 \times 10^4 \text{ J}$ per 1 min?

Hint: In solving problems 329–331 assume that solar energy is not absorbed by the Earth's atmosphere.

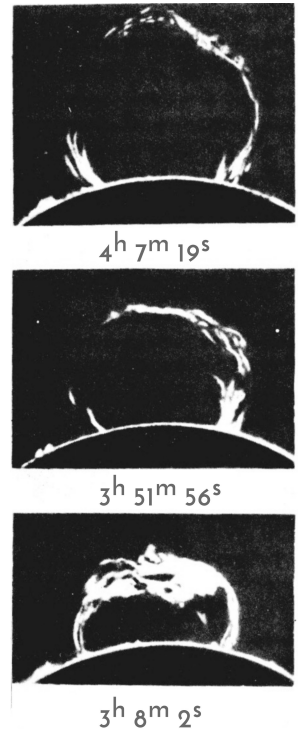
4 *The Sun and Stars*



Figure 19: The Sun with spots and prominences.

4.2 Tasks for Independent Work

333. Using a photograph of the Sun (see the colour illustration on the inside back cover of the textbook (Figure 19)), determine with a ruler the size of the largest spot and the size of the group of spots in comparison with the diameter of the Earth, knowing the angular diameter of the Sun ($32'$) and its parallax ($8.8''$).
334. Using a photograph of the Sun (see the colour illustration on the inside back cover of the textbook, Figure 19 here), sketch the approximate position of its equator and axis of rotation, making use of the known positions of sunspots.
335. Determine the height of the arch and the speed of rise of a prominence (in km/s) by measuring its positions on three photographs (Figure 20-Figure 22). From the three images in Figure 20, determine whether the motion of this prominence is uniform. *Hint:* As a scale, determine the radius of the Sun from its visible segment in the image. This can be done more accurately by drawing circles of different diameters on tracing paper and overlaying them on the segment to find the best fit for the given photograph.



4 The Sun and Stars

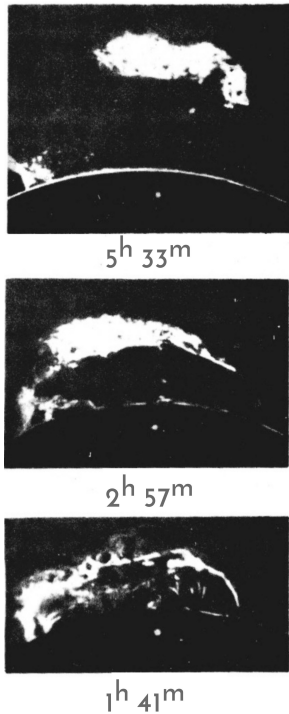


Figure 21: Three phases of the rise of a prominence arch. The white dot represents the Earth at the same scale.

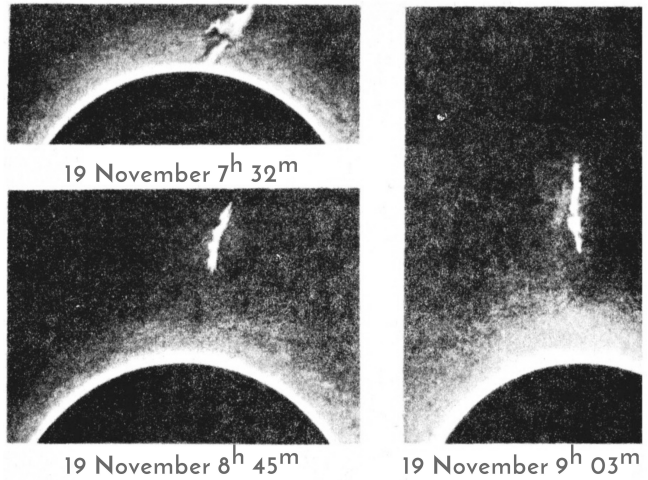


Figure 22: Successive phases of a prominence ejection.

336. From the drawings (Figure 23) of the successive positions of two groups of spots (each shown as a single point) on the solar disk, determine the synodic rotation period of the Sun. Estimate the approximate heliographic latitudes of these two groups, keeping in mind the spherical nature of the Sun.

4.2 Tasks for Independent Work

Instruction: By approximately marking the equator of the Sun in the figure and drawing a chord parallel to it, representing the displacement of the spot due to the Sun's rotation, take half of its length as the radius of the circle described by the spot during the Sun's rotation. The linear distance of the spot from the middle of the chord is equal to $r \sin \theta$, where θ is the angle that the spot has travelled from the central meridian to the given visible position of the spot. In this way one can determine the angular velocity of the Sun's rotation, and from it the period of its revolution.

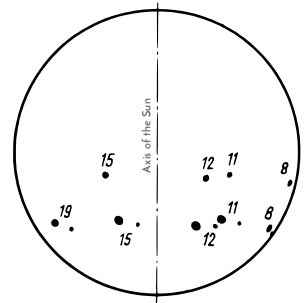


Figure 23: On the image of the solar disc are drawn the successive positions of two groups of spots on the following days of April: the 8th at 9^h00^m, the 11th at 10^h30^m, the 12th at 7^h10^m, the 15th at 12^h30^m, and the 19th at 9^h30^m.

337. Determine the period of the Sun's synodic rotation from Figure 79 (Figure 24 here) of the textbook, using the indication given for the previous problem.

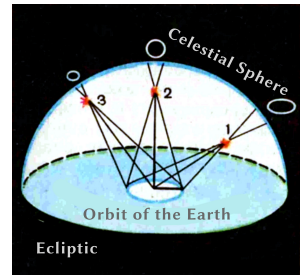


Figure 24: The annual parallaxes of the stars.

4.3 Nature of Stars

Standard Problems

1. The parallax of the star Vega is approximately $0.1''$. What is its distance in parsecs, and how long does its light travel to reach the Earth? And to the Sun?
Solution: The distance D in parsecs to a star outside the Solar System is inversely proportional to its annual parallax p , i.e.

$$D = \frac{1}{p}.$$

Hence, $D = 10$ parsec. Since 1 parsec ≈ 3.26 lightyear, the light from Vega takes about

$$10 \times 3.26 = 3.26 \text{ lightyear}$$

to reach the Earth.

The distance from Earth to the Sun is traversed by light in 8 min, so the distances from Vega to the Earth and from Vega to the Sun may be considered practically the same. They differ by less than 8 min, whereas the uncertainty of distance determination is of the order of one year. Compared with this, 8 min is negligible.

4.3 Nature of Stars

2. How many years would one need to travel at the speed of light for Vega to become twice as close? And if travelling at 30 km/s?

Solution. At the speed of light, the time is $32.6/2 = 16.3$ year. The speed 30 km/s is about 10^4 times smaller than the speed of light. Therefore, the travel time must be 10^4 times longer, i.e. $16.3 \times 10^4 \approx 1.63 \times 10^5$ year.

3. What is the luminosity L of the star ζ Scorpii if its apparent magnitude is 3, and its distance is 7500 ly?

Solution. It is known that the luminosity L of a star can be determined from its absolute magnitude M :

$$\log L = 0.4(5 - M).$$

The absolute magnitude M can be found from the known apparent magnitude m and the distance D (in parsecs):

$$M = m + 5 - 5 \log D.$$

Converting the given distance into parsecs:

$$D = \frac{7500 \text{ ly}}{3.26 \text{ ly/pc}} \approx 2300 \text{ pc}.$$

4 The Sun and Stars

Then

$$M = 3 + 5 - 5 \log(2.3 \times 10^3) \approx -8.8,$$

which gives

$$L \approx 3.3 \times 10^5 L_{\odot}.$$

4. By how many times is a certain star larger in diameter than the Sun if its luminosity is 100 times greater, while their temperatures are the same?
Solution. Since the temperatures of the stars are equal, their luminosities are proportional to the squares of their radii:

$$\frac{L}{L_{\odot}} = \left(\frac{R}{R_{\odot}}\right)^2 \cdot \left(\frac{T}{T_{\odot}}\right)^4 = 100.$$

Hence,

$$R = R_{\odot} \sqrt{100} = 10R_{\odot}.$$

5. Determine the average density of a red supergiant star in g/cm^3 if its diameter is 300 times greater, and its mass is 30 times greater than the Sun.
Solution. The mean density is given by

$$\rho = \frac{M}{V}.$$

4.3 Nature of Stars

For the red supergiant,

$$\frac{\rho}{\rho_{\odot}} = \frac{M/M_{\odot}}{(R/R_{\odot})^3} = \frac{30}{300^3}.$$

Since $\rho_{\odot} \approx 1.4 \text{ g/cm}^3$,

$$\rho \approx 1.4 \cdot \frac{30}{27 \times 10^6} \approx 1.5 \times 10^{-6} \text{ g/cm}^3.$$

338. The probable error of Rigel's parallax $0.006''$ is $\pm 0.006''$. What can be said about the star's distance?
339. Sirius B (the companion of Sirius) has a diameter 30 times smaller than the Sun's, and its mass is 0.9 times the solar mass. Compute its mean density, given the Sun's mean density is 1400 kg/m^3 .
340. The parallax of the star 61 Cygni is $0.37''$. What is its distance in light-years?
341. Determine the luminosity of a star whose surface temperature equals the Sun's but whose radius is 10 times the solar radius.
342. In novae, the brightness usually increases at roughly constant temperature due to photospheric expansion. If the change in apparent magnitude is 8, by what factor did the star's radius change?
343. By what factor did the apparent brightness of Nova Aquilae increase if before the outburst it had $m =$

4 The Sun and Stars

- +10.5 and during the outburst it reached $m = +1.0$?
344. What is the ratio of component radii in an Algol-type eclipsing binary if the eclipse is central, the companion is dark, and the ratio of maximum to minimum brightness is 2?
345. By what factor is Arcturus closer than Deneb, if their parallaxes are $p = 0.085''$ and $p = 0.005''$, respectively?
346. The parallax of Sirius is $0.37''$, and the parallax of Spica is $0.02''$. Express their distances in parsecs, light-years, astronomical units, and kilometers.
347. By what factor is Proxima Centauri (for which $p = 0.76''$, $m = 10.5$) less luminous than the Sun?
348. Which object—the Sun or S Doradus (with absolute magnitude $M = -9.4$) has the greater luminosity, and by what factor?
349. Compute how many times Rigel is more luminous than the Sun, given its parallax $0.0069''$ and apparent magnitude 0.34.
350. Determine the absolute magnitudes of the components of Krüger 60, given their apparent magnitudes $m_1 = 9.6$ and $m_2 = 11.4$, and parallax $0.257''$.
351. Express the luminosities of the binary components of Krüger 60 relative to the Sun, given their abso-

4.3 Nature of Stars

- lute magnitudes $M_1 = 11.6$ and $M_2 = 13.4$, and the Sun's absolute magnitude $M_{\odot} = 4.85$.
352. The apparent magnitude of Sirius is $m_1 = -1.58$ and that of its companion is $m_2 = 8.44$. By what factor is Sirius's intrinsic brightness larger than that of its companion? *Hint:* The stars are essentially at the same distance.
353. Compute the absolute magnitude of Sirius, given parallax $0.371''$ and apparent magnitude $m = -1.58$.
354. Determine the absolute magnitude of Antares, given parallax $0.009''$ and apparent magnitude $m = +1.22$.
355. The parallax of Altair is $0.20''$ and that of Vega is $0.12''$. Express their distances in pc, ly, au, and km.
356. A star has parallax $0.312''$ with probable error $\pm 0.006''$. What can you conclude about its distance?
357. From the following [Table 1](#) of some nearby stars, select several of different types.
- (a) Using the tabulated apparent magnitudes and distances (or parallaxes), compute their absolute magnitudes M and luminosities L . Plot the results on a "color-luminosity (H-R) diagram".
- Hint:* Compute M and L and place the points

4 The Sun and Stars

on the diagram (prepare a tracing of the diagram from the textbook's Figure 90 ([Figure 25](#) here)).

- (b) Estimate the masses of the same stars from their luminosities.
- (c) State the color of each star from its spectral type listed in the table.

Table 1: Properties of some nearby stars.

Star	Magnitude	Spectral Type	Distance (pc)
Proxima Centauri	11.0	M5	1.32
Alpha Centauri A	0.3	G2	1.34
Wolf 359	13.5	M8	2.3
Sirius A	-1.6	A1	2.66
Sirius B (companion)	8.7	A5	2.66
Procyon A	0.5	F5	3.5
Procyon B (companion)	10.7	F5	3.5
Epsilon Eridani	3.7	K2	3.3
Vega	0.1	A0	8.0
Bellatrix	1.6	B2	40
Zeta Orionis	1.8	O9	45
Spica	1.2	B1	50
Betelgeuse	0.9	M2	200

4.3 Nature of Stars

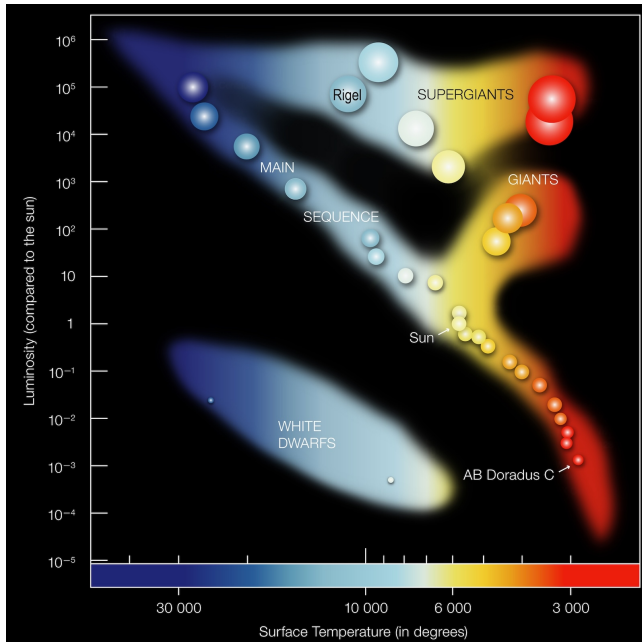


Figure 25: The H-R diagram.
From [Wikipedia](#) CC BY-SA 4.0.

4.4 Double Stars. Stellar Masses

Typical Problems

1. For a binary star, the parallax is $p = 0.05''$, the semi-major axis of the apparent orbit is $a = 2''$, and the orbital period of the components is 100 years. Find the sum of the stellar masses, as well as the individual masses of each star, if the stars are displaced from the centre of mass in the ratio 4 : 1.

Solution. The linear semi-major axis is

$$A = \frac{a}{p} = \frac{2''}{0.05''} = 40 \text{ au.}$$

According to Kepler's third law:

$$m_1 + m_2 = \frac{A^3}{T^2} = \frac{40^3}{100^2} = 6.4 M_{\odot}.$$

Since the distances to the centre of mass are in the ratio $m_1 : m_2 = 4 : 1$, let $m_1 = 4m_2$. Then

$$m_1 + m_2 = 5m_2 = 6.4 M_{\odot},$$

hence

$$m_2 = 1.3 M_{\odot}, \quad m_1 = 5.1 M_{\odot}.$$

4.4 Double Stars. Stellar Masses

2. Suppose that instead of Earth, another Sun were orbiting around the Sun. What would be the orbital period of such a system?

Solution. By Kepler's third law:

$$m_1 + m_2 = \frac{A^3}{T^2}.$$

Taking the solar mass as 1, one year as 1, and the semi-major axis $A = 1$ au, we have:

$$1 + 1 = \frac{1}{T^2}.$$

Thus,

$$T = \frac{1}{\sqrt{2}} \approx 0.707 \text{ years}.$$

358. Calculate the sum of the masses of the binary star Capella, if the semi-major axis of its orbit is equal to 0.85 au and the orbital period is 0.285 years.
359. Determine the sum of the masses of the binary star Procyon, whose companion has an orbital period of 39 years and a semi-major axis of 13.0 au.
360. Calculate the mass of the binary star α Centauri, which has a parallax of $0.75''$, an orbital period of 79 years, and its semi-major axis is observed from Earth under an angle of $17'36''$.

4 The Sun and Stars

361. The binary star α Gemini has a parallax of $0.076''$, an apparent angular size of the semi-major axis of $6.06''$, and an orbital period of 306 days. Determine the sum of the masses of this binary star.
362. The parallax of the binary star χ Pegasi is determined with a possible error of $\pm 0.005''$. How would the calculated sum of the masses of this pair of stars change if the parallax were taken to be larger than given, by the value of this possible error?
363. What is the maximum apparent separation of the Earth from the Sun, if observed from the nearest star, α Centauri, whose parallax is $0.75''$? Could it be seen from there separately from the Sun through a telescope with an objective diameter of 1 m (assuming that the issue of the apparent brightness of the Earth from there plays no role)?
364. By choosing one of the figures ([Figure 26-Figure 33](#)), estimate the orbital period of a binary star, the orientation of the projection of the major axis on the sky, knowing that it passes through the centre of the ellipse and through the primary star. Estimate the eccentricity of the apparent orbit (equal to the eccentricity of the true orbit). Determine the angular size of the semi-major axis. Calculate the length of the semi-major axis (in au) and the sum of the masses of the components for the stars whose

4.4 Double Stars. Stellar Masses

parallaxes are given in the table on the right.

365. In (Figure 26-Figure 33) are shown the observed orbits of some visual binary stars. Most of them have such long orbital periods that over decades, even centuries of observations, the stars have not yet managed to complete their orbits. One may notice from the positions of the plotted observational points that, over time, the measurements become more accurate and better fit the proper ellipses, which the orbits of binary stars actually are. Astronomers, as a rule, have to sketch the ellipses of orbits from incomplete cycles of observations. We suggest that you examine how the orbital ellipses are completed, so that one can estimate the orbital periods of stars and the sizes of their orbits, for statistical purposes and for determining the stellar masses, which can be done reliably only in the case of binary stars.

Star	Parallax
Sirius	0.37";
Castor	0.072";
5 Herculis	0.11";
ζ Ursae Majoris	0.146".

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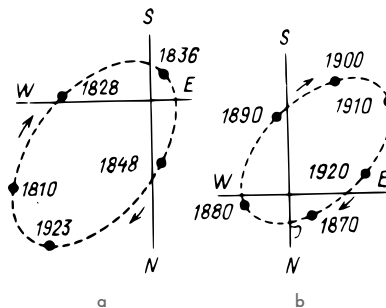


Figure 26: The orbits of a) γ Virginis, b) χ Ursae Majoris.

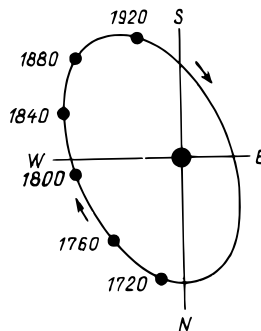


Figure 27: The orbit of Castor.

4.4 Double Stars. Stellar Masses

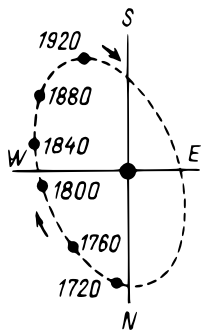


Figure 28: The orbit of the companion of ζ Herculis.

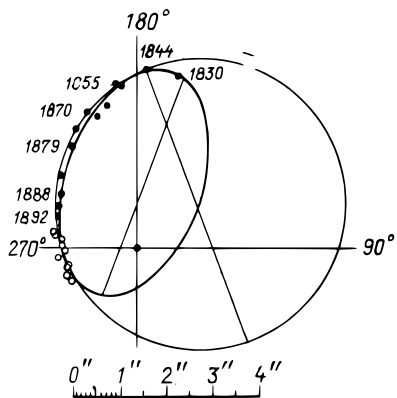


Figure 29: The orbit of Σ 1785.

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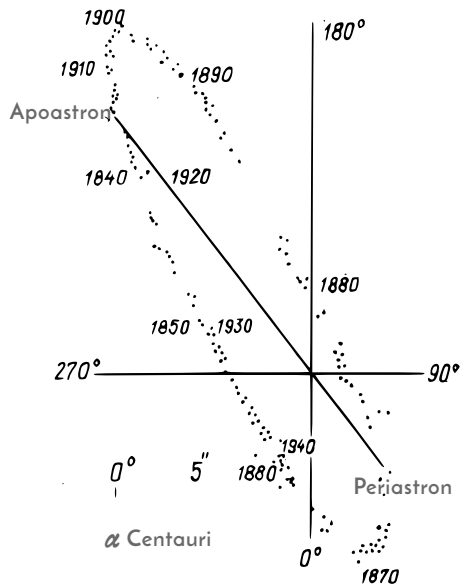


Figure 30: The orbit of α Centauri.

4.4 Double Stars. Stellar Masses

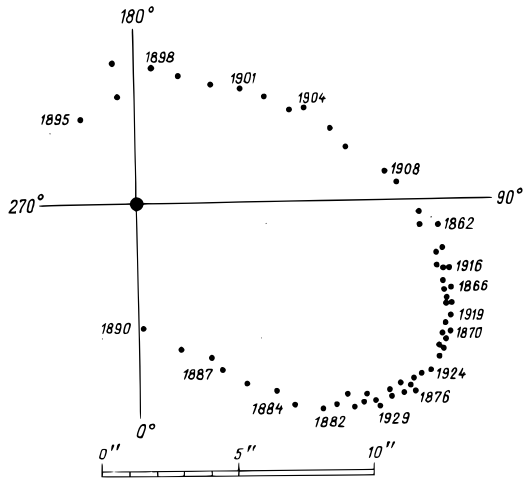


Figure 31: The orbit of Sirius's companion.

4 The Sun and Stars

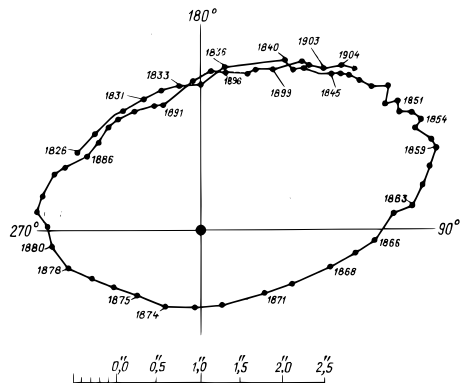


Figure 32: The orbit of ξ Ursae Majoris.

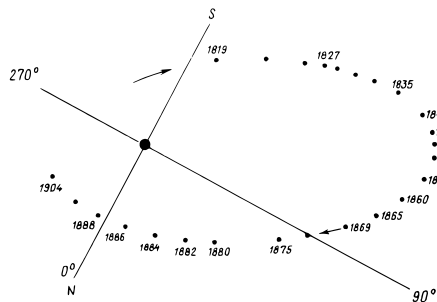


Figure 33: The orbit of the star 70 Ophiuchi.

4.5 Variable And Novae Stars

366. Determine the period of variation in the brightness of the eclipsing binary star (Algol type) RW Ophiuchi (to the accuracy of five decimal places), if some of the minima were observed at the following moments, expressed in so-called Julian days⁴:

2416604.701	2418112.739
6641.572	8138.548
7334.753	8477.764

⁴ Julian days are counted from a very distant moment, so the time elapsed between two calendar dates is found by simply subtracting their numbers.

Note: In the intervals between the specified minima, the period must be contained an integer number of times.

367. Using the data given below (see [Table 2](#)), plot the graph of the variation in the brightness of the variable star S Sagittae with time. From it, determine the period of this variable star.

Notes.

- (1) In the left column are given the day numbers counted from some starting point, and the moment in fractions of a day; in the right column is the measured stellar magnitude.
- (2) Breaks between observations are caused by cloudy weather.

4 The Sun and Stars

- (3) The recommended scale of the graph is as follows: 1 day = 5 mm, 1 stellar magnitude = 20 mm.

Date, days	m	Date, days	m	Date, days	m	Date, days	m
858.30	6.22	864.21	5.51	885.35	6.22	895.29	5.01
859.25	6.31	865.27	6.01	887.30	5.18	895.34	5.10
860.27	6.21	867.32	6.30	888.34	5.30	895.38	5.13
861.20	5.50	870.32	5.09	889.35	5.34	896.35	5.22
861.25	5.42	871.40	5.32	891.27	5.84	897.40	5.34
861.43	5.14	881.23	6.22	892.37	6.35	898.40	5.75
862.30	5.30	882.30	5.95	893.35	6.30	899.40	6.03
863.30	5.40	883.33	6.13	894.35	5.90	900.25	6.25

Table 2: Magnitude data for variable star *S Sagittae*. Plot these data on a graph and determine the probable type of the given variable star, as well as its approximate period and amplitude.

368. Determine the mean period and amplitude of brightness variation of χ Cygni from [Figure 34](#).
369. From [Figure 35](#) determine (more precisely) the moments of maxima of the variable star and the period of its brightness variation.
370. In May the variable star had the following brightness (see [Table 3](#)). Plot these data on a graph and determine, in all probability, the type of the given variable star and approximately its period and amplitude.
371. Plot the following observations of the variable star

4.5 Variable And Novae Stars

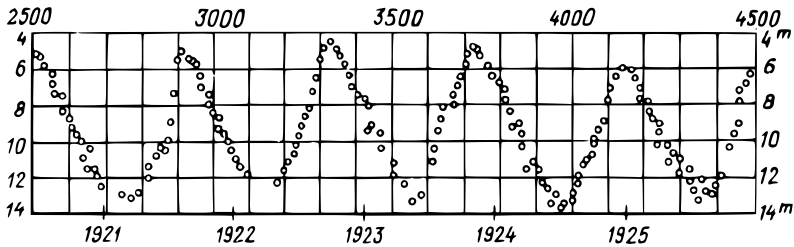


Figure 34: The mean period and amplitude of brightness variation of χ Cygni. The numbers at the top are Julian days, and at the bottom are the years.

Table 3: Magnitude data for a variable star.

Date	<i>m</i>	Date	<i>m</i>	Date	<i>m</i>	Date	<i>m</i>
4	4.00	14	4.13	21	3.72	25	4.16
6	3.87	16	3.70	22	3.84	28	3.93
7	3.91	17	3.90	23	3.97	29	4.10
13	4.05	19	4.11	24	4.12	31	4.08

on a graph (see [Table 4](#)).

372. Using the “period–absolute magnitude” curve ([Figure 36](#)) found for Cepheids, determine the distance to the Cepheid in ζ Gemini, which has a period of 10 days and an average apparent photographic magnitude of 4.8.

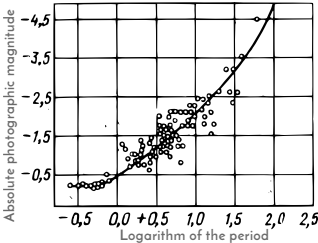


Figure 36: The period-absolute magnitude curve of Cepheid variables. The abscissa axis shows the logarithms of the period, expressed in days.

373. The variable star δ Cephei has a period of 5 days

4 The Sun and Stars

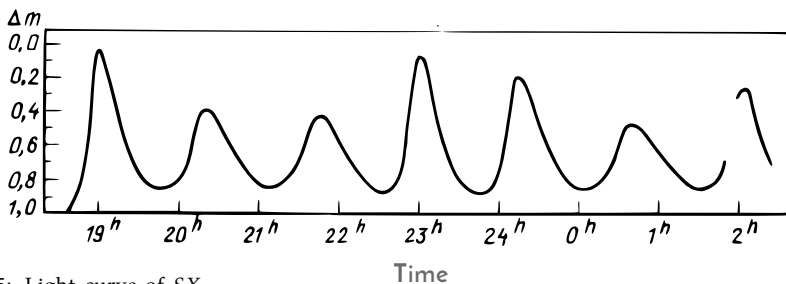


Figure 35: Light curve of SX Phoenixis.

Table 4: Magnitude data for a variable star. The first four digits of the date are repeated and may be omitted after the first line.

Date	m	Date	m	Date	m	Date	m	Date	m
2426423	6.98	461	6.96	480	7.52	498	7.44	539	7.53
436	7.17	462	6.95	482	7.65	513	7.59	540	7.60
440	7.34	464	6.86	484	7.69	514	7.65	545	7.45
448	7.36	467	6.88	486	7.57	515	7.61	555	7.34
454	7.23	472	6.99	488	7.23	516	7.77	557	7.35
457	6.90	473	7.06	492	7.49	526	7.44	560	7.05

and an average apparent photographic magnitude of 4.4. At what distance from us (in parsecs) is δ Cephei located?

Hint: Use Figure 36.

374. Determine the ratio of the surface brightnesses of the components of the binary star Algol, if their

4.5 Variable And Novae Stars

brightness is in the ratio $0.93 : 0.07$, and their radii are 0.21 and 0.24 of the Sun's radius.

375. The variable star Mira Ceti (the Wonderful Star of the Whale) at maximum brightness reaches magnitude 2.5, and at minimum brightness 9.2. How many times brighter is it at maximum than at minimum?
376. How many times fainter is a variable star at minimum brightness than at maximum, if at maximum it reaches magnitude 9.5, and at minimum 12.5?
377. The maximum of the star χ Cygni was on May 20, and its period is 405.6 days. When were its maxima in the three following years?

Hint: It is convenient to use Julian days, counting years as exactly 365 days (see the hint to Problem 366).

378. One of the minima of Algol occurred on 3 January at $13^{\text{h}}55^{\text{m}}$. Given that the period of Algol is 2.8673 days, compute the moments of the nearest minima in the same year and month.

Hint. For calculations it is convenient to construct a table in which fractions of days are expressed in hours and minutes.

379. By how much does the radius of a Cepheid vary if the amplitude of its brightness variation is 1.5 magnitudes, assuming that the surface brightness

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remains constant?

380. A new star in Perseus in 1901 increased its brightness over two days from the 12th to the 2nd magnitude. How many times brighter did it become on average per day?
381. The nova of 1918 in the constellation of Aquila at maximum brightness had an absolute magnitude $M = -8.8$. How many times brighter was it than the Sun? At what distance was it located if its apparent magnitude was $m = -1.1$? From what distance would it appear as bright as the full Moon?

Hint: The apparent magnitude of the full Moon is $m = -12.5$.

382. In the constellation of Hercules in 1934, a nova erupted. Its brightness increased by 5 magnitudes within one day. The distance to the star, measured immediately after the outburst, turned out to be 1800 light-years. Based on the principle that no velocities can exceed the speed of light, prove that the mentioned outburst could not have been caused by a sudden approach of the star to the Earth.
383. In the spectrum of the nova of 1934 in Hercules, dark lines were shifted relative to their normal positions toward the violet end. The H_γ line ($\lambda = 4341 \text{ \AA}$) was shifted by $10.1 \text{ \AA}$. What was the veloc-

4.5 Variable And Novae Stars

ity of the gas ejected from the star, which caused the appearance of these absorption lines in the spectrum?

5 Structure Of The Universe

5.1 Motions of Stars

Note. The curve of dependence of the period on the absolute stellar magnitude, required in some problems, is given in [Figure 36](#).

Typical Problems

1. The proper motion of a star is $\mu = 0''.1$ per year. Its distance is $D = 10$ pc. What are its tangential and spatial velocities, if its radial velocity is $V_r = 10$ km/s?

Solution: The tangential velocity is

$$V_\tau = 4.74 \mu D, \quad V_\tau = 4.74 \cdot 0.1 \cdot 10, \quad V_\tau = 4.74 \text{ km/s.}$$

Rounding the answer to 5 km/s, we find the spatial velocity:

$$V = \sqrt{V_r^2 + V_\tau^2}, \quad V = \sqrt{100 + 25} \approx 11 \text{ km/s.}$$

5 Structure Of The Universe

2. Determine the distance to a globular star cluster if several short-period Cepheids with apparent magnitude $m = 15.5$ and absolute magnitude $M = 0.5$ are observed in it.

Solution: We find the distance D using the formula:

$$m - M = 5 \log D - 5,$$

hence

$$5 \log D = 15.5 + 5 - 0.5 = 20.0,$$

therefore

$$\log D = 4, \quad D = 10^4 \text{ pc.}$$

3. What is the linear size of the cluster described in the previous problem, if its angular diameter is $d = 3'$? What apparent magnitude would our Sun, with absolute magnitude $M = 5$, have at this distance?

Solution. The linear diameter of the cluster is given by

$$2R = \frac{d'' \cdot D}{206265''},$$

where d is the angular diameter in arcseconds, and D is the distance in parsecs. Since $d = 3' = 180''$

5.1 Motions of Stars

and $D = 10^4$ pc, we obtain

$$2R = \frac{180'' \cdot 10^4 \text{ pc}}{2 \times 10^5''} \approx 9 \text{ pc}.$$

For Cepheids in this cluster, the difference between apparent and absolute magnitude was found to be $m - M = 15$. Hence the apparent magnitude of the Sun at this distance would be $m = M + 15 = 5 + 15 = 20$. Thus, the Sun would only be visible through the largest telescopes in the world.

4. In a galaxy with a redshift corresponding to a recession velocity of $V = 10,000$ km/s, a supernova appeared with apparent magnitude $m = 18$. How far away is it, what is its absolute magnitude M , and what is its luminosity L ?

Solution. With Hubble's constant $H = 100$ km/(s Mpc), the distance to the galaxy is

$$D = \frac{V}{H} = \frac{10000}{100} = 100 \text{ Mpc} = 10^8 \text{ pc}.$$

Using the distance modulus:

$$M = m + 5 - 5 \log D.$$

Substituting values:

$$M = 18 + 5 - 5 \log(10^8) = 23 - 40 = -17.$$

5 Structure Of The Universe

The luminosity is found from the relation

$$\log L = 0.4(5 - M).$$

Substituting $M = -17$:

$$\log L = 0.4(5 - (-17)) = 0.4 \cdot 22 = 8.8,$$

$$L \approx 6.3 \times 10^8 L_{\odot}.$$

384. A star moves in space with a velocity of 50 km/s towards the observer at an angle of 30° to the line of sight. Determine the radial and tangential components of the star's velocity.
385. How far will the Solar System travel in one year in the direction of the apex of its motion if its velocity is 19.5 km/s?
386. In which part of the sky, due to the motion of the Solar System in space, do the apparent angular distances between stars continuously increase, and in which part do they continuously decrease?
387. The distance to Sirius is 2.70 pc, but due to its motion Sirius is approaching at a rate of 8 km/s. Calculate after how many years the apparent brightness of Sirius will increase by a factor of two.
388. If a star lies on the ecliptic, what was the difference between the observed radial velocities on 9 September 1979 and 7 March of the following year?

5.1 Motions of Stars

389. Knowing that the equatorial coordinates of the apex of the Solar System's motion are $\alpha = 270^\circ$ and $\delta = +30.5^\circ$, determine from a star chart which constellation the apex lies in.
390. The radial velocity of the star Aldebaran is $V_r = +54 \text{ km/s}$, its proper motion is $\mu = 0.20''/\text{year}$, and its parallax is $p = 0.05''$. Determine the full space velocity of Aldebaran.
391. The radial velocity of Arcturus is $V_r = 22 \text{ km/s}$, and the tangential velocity is $V_\tau = 23 \text{ km/s}$. Find the angle formed between the star's direction of motion and the line of sight.
392. The radial velocity of Betelgeuse is $V_r = +21 \text{ km/s}$, its proper motion is $\mu = 0.032''/\text{year}$, and its parallax is $p = 0.012''$. Determine the full space velocity of the star relative to the Sun and the angle formed between its direction of motion in space and the line of sight.
393. Assuming that in the local galactic motion the Sun moves with a velocity of 20 km/s towards the star Vega, compute the time it will take for the Sun to reach the region of space now occupied by Vega, if Vega is currently 27 light years away.
394. The total space velocity of β of the Southern Cross is $v = 21 \text{ km/s}$, and the radial velocity is $V_r = +13 \text{ km/s}$. Determine the tangential velocity.

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- 395. The total space velocity of Canopus is 23 km/s and it makes an angle of 37° with the line of sight. Determine the radial and tangential components of the velocity.
- 396. The direction of motion of Capella forms an angle of 48.2° with the line of sight, and its total space velocity is 45 km/s. Determine the annual proper motion of the star, if its parallax is $0.063''$.
- 397. Barnard's "flying star" has an annual proper motion of $10.25''$ and a parallax of $0.546''$. What is its tangential velocity?
- 398. What is the tangential velocity of Sirius, if its parallax is $0.371''$ and its annual proper motion is $1.315''$?
- 399. The open star cluster Pleiades contains about 160 stars. Assuming the cluster has a spherical shape, determine its average stellar density (number of stars per 1 pc^3), taking the cluster's diameter to be 7 pc.
- 400. The planetary nebula in the constellation Lyra has an angular diameter of $83''$ and is located at a distance of 660 pc from us. What are its linear dimensions in astronomical units?
- 401. How long would one have to wait for an answer to a radio message sent to the Andromeda galaxy, if its distance is 620 000 pc?

5.1 Motions of Stars

402. In the globular star cluster NGC 5694, the apparent magnitudes of stars are greater than their absolute magnitudes by 18. What is the distance to this star cluster?
403. The angular diameter of the globular star cluster NGC 5694 is $3'$, and its distance is 40 000 pc. What is the linear diameter of the cluster?
404. The apparent magnitude of a Cepheid in a star cluster in Hercules is $m = 15.1$. From the period of its variability, it is known that the absolute magnitude of the Cepheid is $M = -9.2$. Determine the distance to the cluster in Hercules.
405. The star cluster in Hercules is located 10.5 kpc away, its angular diameter is $12'$, and its total apparent magnitude is 5.9. Compute the actual diameter of the cluster and its absolute magnitude.
406. Using the “period–absolute magnitude” relation for Cepheids (see [Figure 36](#)), determine the distance (in light years) and the linear diameter of the spiral galaxy in the constellation Triangulum, if its angular diameter is 1° , and the observed Cepheids have a period of 13 days with an apparent magnitude of $m = 19.6$.
407. Assuming the Hubble constant $H = 100 \text{ km}/(\text{s Mpc})$, estimate the distance to a galaxy if the redshift in its spectrum corresponds to 10^4 km/s .

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408. The average apparent photographic magnitude of short-period Cepheids (with an average period of 0.54 days), located in the globular star cluster Messier 3, is $m = 15.50$. Using the “period–absolute magnitude” curve (Figure 36), determine the distance to this globular cluster.
409. Compute by how many times the number of stars brighter than an apparent magnitude m must be smaller than the number of stars down to magnitude $(m + 2)$, assuming that stars are uniformly distributed in space and all have the same intrinsic brightness.
410. Two novae, one very bright and the other very faint, are located at the same apparent angular distance from the mid-plane of the Milky Way (i.e. they have the same galactic latitude). Are these stars equally distant in linear measure from the plane of the Milky Way? *Note.* The absolute magnitudes of all novae at maximum brightness do not differ very much.
411. Using a star map, determine through which constellations the Milky Way passes, and in which constellations it intersects the celestial equator.
412. Draw a diagram and show that the fact that the central line of the Milky Way deviates somewhat from a great circle (being a small circle passing at

a small distance from the great circle) indicates that the Solar System is not located exactly in the galactic plane (the plane of symmetry of the stellar system of the Milky Way).

413. What will the Milky Way look like to an observer situated in its plane of symmetry but very far from its centre? How will the observed form of the Milky Way be affected if the observer is located far outside its plane of symmetry?

Hint. Neglect light absorption in space.

414. In a spiral galaxy located 7.5 million light years away, two Cepheids have been discovered with identical periods. One of them is located at the edge of the galaxy nearest to us, and the other is 20,000 light years farther away. What is the difference in their apparent magnitudes, assuming their absolute magnitudes are exactly the same?
415. If the absolute photographic magnitude of some galaxies is $M = -13.8$, and the limiting photographic apparent magnitude for a 2.5-metre reflector is $m = 20.2$, then what is the distance (in light years) to the most distant of these galaxies that are observable with this telescope?
416. By the “Cepheid method” the distances to a number of spiral galaxies were determined. Later it was discovered that interstellar dust somewhat ab-

5 Structure Of The Universe

sorbs the light passing through it. How should this discovery have affected the previously calculated diameters of spiral galaxies?

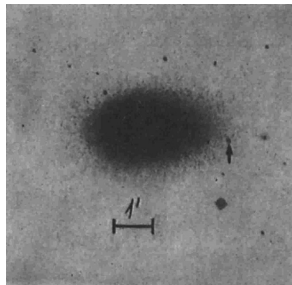
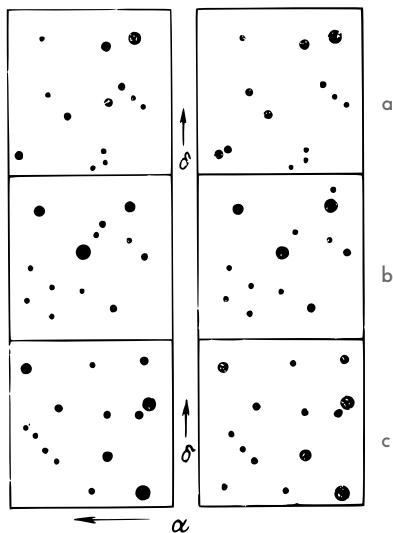


Figure 37: A photograph (the negative) of a distant stellar system – a galaxy with a supernova indicated by an arrow (the angular scale of the photograph is indicated).

417. At what distance from the centre of a galaxy is its supernova star located (see photograph 87 (Figure 37 here) in the textbook), if the apparent diameter of the galaxy is $2'$ and its distance from us is 10^7 pc?
418. If we assume that the linear diameters and absolute total magnitudes of all globular clusters are the same, then by what formula should their apparent total magnitudes m and their apparent angular diameters d be related, given that their distances are different?
419. How will the relationship between the angular diameters and the total apparent magnitudes of globular clusters (see the previous problem) change if there is significant absorption of light in interstellar space?
420. Compare a pair of photographs (Figure 38 and Figure 39) of the same region of the sky taken 50 years apart, and identify a nearby star with large proper motion. Estimate its proper motion in arcseconds per year, given that the scale of the photograph is $1''$ per cm. (The right-hand images

5.1 Motions of Stars



were taken later than the left-hand ones.)

Figure 38: Paired (horizontally) images, on each of which there is one fast-moving star. Scale 1" per 1 cm.

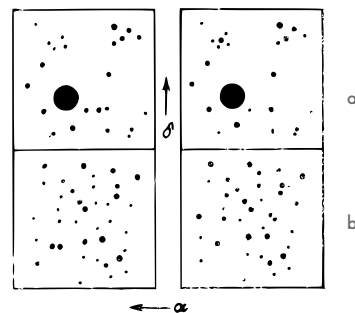


Figure 39: Find a moving star.

6 Solutions

6.1 Introduction

Answers to many of the problems can be found in the text and in the tables of the textbook. Get used to using them as references.

13. $2.512^{23} = 1.6 \times 10^9$ times.
14. $6.3 \cdot 10^2$ times.
15. 4 times, i.e. by $1.5m$.
16. 100, 40, 16, 6.3
17. About 1100.
18. By $0.7m$.
22. At 8 p.m.
23. ≈ 1.9 m.
24. 3 km.
25. $r = \sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos \theta}$.
26. During daily rotation and throughout the year, the Earth's axis maintains an unchanged direction among the stars, i.e. the celestial poles remain

6 Solutions

- fixed among the stars.
27. $34^{\circ}15'$.
 28. 55° .
 29. 50; 80; 70; 40; 20; 50° .
 30. Either the observer is at one of the Earth's poles, or the celestial body is at one of the celestial poles.
 31. Accurate to $8''$.
 32. $\delta = \varphi$
 33. $\delta > (90^{\circ} - \varphi)$; $\delta < (90^{\circ} - \varphi)$.
 34. $55^{\circ}45'$; $50^{\circ}27'$; $41^{\circ}43'$
 35. Until $\delta = -30^{\circ}3'$; until $\delta = -48^{\circ}40'$.
 36. About 11 June or about 3 July.
 37. $\delta = 0$.
 38. $35^{\circ}28'$.
 40. $17^{\circ}3'$.
 41. $14^{\circ}3'$; $4^{\circ}34'$.
 42. $-16^{\circ}36'$.
 43. $\delta = 90^{\circ} - \varphi$
 44. No.
 47. Above the horizon are only stars of the southern hemisphere. In diurnal motion, all stars neither rise nor set, but describe circles parallel to the horizon.
 48. All of them are inclined to the horizon, and at the equator the diurnal paths of the stars are perpendicular to the horizon.

6.1 Introduction

49. $51^{\circ}2'$.
50. In Odessa and Tbilisi – not always. In Moscow and Kiev – always.
51. No.
52. Starting from latitude 35° /
53. $33^{\circ}13'$. Around 1 May.
54. Rising between north and east, setting between north and west; rising between south and east, setting between south and west.
55. Two times; one time.
56. March 21 and September 23.
57. Half.
59. $10^{\circ}45'$, $26^{\circ}22'$.
60. About $71^{\circ}26'$, $85^{\circ}31'$. It changes slightly because the calendar year is not equal to the period of the Sun's revolution along the ecliptic. In addition, the Sun's axis changes during the day.
61. 35° .
63. $23^{\circ}27'$.
64. $66^{\circ}33'$.
65. From May 17 to July 28; from November 18 to January 25.
66. $\varphi = 40^{\circ}$, $\varepsilon = 23^{\circ}27'$.
67. $\varphi = 34^{\circ}47'$, $\varepsilon = 23^{\circ}54'$.
68. $89^{\circ}2'$.
69. 180; 0; 270; 90° .

6 Solutions

- 70. 42° .
- 71. $66^\circ 33'$.
- 72. $123^\circ 47' 45''$, 200° , $212^\circ 30'$, $312^\circ 30'$, 359° .
- 73. $\alpha = 18^{\text{h}} 00^{\text{m}}$, $\delta = 66.5^\circ$.
- 74. 45° .
- 75. In the constellation of Gemini.
- 76. $\alpha = \delta = 0$.
- 77. For the point of the vernal equinox.
- 78. a) Taurus; b) Scorpius; c) Cygnus. a) Aldebaran; b) Antares; c) Deneb.
- 81. For the first: $18^{\text{h}} 57^{\text{m}} 1^{\text{s}}$, 13.
- 82. For the first: $49^\circ 17' 15''$.
- 83. 5° .
- 84. In the southwest.
- 85. At the Earth's equator.
- 88. $\alpha = 0^{\text{h}} 16^{\text{m}} 45^{\text{s}}$, $\delta = -0^\circ 6.2'$.
- 89. $57^\circ 42'$.
- 90. $2^\circ 0'$.
- 91. $53^\circ 30'$, $72^\circ 7'$, 90° , $46^\circ 54'$. ($6^\circ 36'$, $25^\circ 13'$, $43^\circ 6'$, 0°).
Tropic of Cancer, Arctic Circle.
- 92. Approximately 12 h.
- 95. After $5^\circ 2'$.
- 96. Approximately at $7^{\text{h}} 20^{\text{m}}$ in the evening.
- 97. At noon around December 22.
- 98. α when the Earth is near perihelion (early January),
 δ around the equinoxes.

6.1 Introduction

- 103. By 2° .
- 104. 0^h and 0° ; 12^h and 0° .
- 105. 6^h , $+23^\circ 27'$; 18^h , $-23^\circ 27'$.
- 106. On the celestial equator; at the east and west points.
- 107. South of 48.5° .
- 108. 60° , 5.
- 109. For all latitudes north of 72° .
- 111. a) It can; b) it cannot.
- 113. $4^h 54^m$, 1.
- 114. $\lambda = 3^h 10^m$ to the west.
- 115. $1^h 22^m$ E.
- 116. $24^h 24^m$.
- 117. About 23^h .
- 118. $3^h 11^m$ E.
- 119. $2^h 1^m$, 2.
- 120. $21^h 36^m$.
- 122. 352 days.
- 123. Because in the tropical year (the duration of the Sun's revolution along the ecliptic) there is not an exact whole number of days.
- 124. 28 February; 9 March; 18 March.
- 125. 28 May at 15h Magadan time.

6.2 Structure of the Solar System

126. No.
127. Superior.
128. Mars is opposite the Sun at opposition; therefore, it is in the constellation of Libra.
129. Venus moves away from the Sun in the sky no further than 45° – 48° , and in the evening must be east of the Sun, i.e., in the constellation Pisces (see star map).
130. Impossible, because Venus does not go farther than 45° – 48° from the Sun.
131. Superior (see answer to Problem 130).
132. 4 au.
133. By a factor of 2.362.
134. 74 years; at a distance of 0.009 au.
135. 1.70 au.
136. 0.63 au.
137. They remain valid when observed from any planet.
138. Using the equation of synodic motion.

$$\frac{1}{S_\Phi} = \frac{1}{T_\Phi} - \frac{1}{T_M}.$$

Hence the synodic period of Phobos $S_\Phi \approx 11.5^{\text{h}}$.
 Number of risings $n = 24.6^{\text{h}} : 11.5^{\text{h}} \approx 2.13$, i.e.,
 Phobos rises twice per day in the west.

6.2 Structure of the Solar System

- 139. 688 days. (Approximate data always give an approximate answer).
- 140. After 780 days
- 141. Either $3/4$ or $3/2$ years.
- 143. 0.72 years.
- 144. No; yes; no.
- 145. 1 day, $6^{\text{h}}48^{\text{m}}$.
- 146. 2 years.
- 147. 7.8×10^7 km; 3.78×10^8 km.
- 148. $R = 1 \text{ au} \cdot \sin 47^\circ = 0.73 \text{ au}$.
- 149. $0.39 \text{ au} = 5.8 \times 10^7 \text{ km}$.
- 150. The period of these phenomena equals the synodic period of Mars, i.e., 780 days.
- 151. 41° .
- 152. About $17'$.
- 153. 29 January of the following year.
- 154. 1 May near α Scorpii. After 2 years, on 1 May, Mars will be in Sagittarius, between the stars λ and π .
- 155. 18 August of the following year.
- 156. $T = 1320 \text{ days}$, $a = 2.35 \text{ au}$.
- 157. 12 September of the following year.
- 158. The acceleration imparted by the Earth to the Moon is greater.
- 160. Equality of centripetal acceleration and free fall acceleration gives. $\omega^2 R = GM/R^2$, $M = 1.9 \times 10^{30} \text{ kg}$.

6 Solutions

- 161. 2.4 km/s.
- 162. Such problems are solved by assuming that the fall time of a planet to the Sun is equal to half of its orbital period around the Sun along a highly elongated ellipse with the major axis equal to the planet's orbital radius. In this case, applying Kepler's third law and taking the orbit as a circle of radius 40 au, we obtain 44 years.
- 163. 17.1 Earth masses.
- 164. 14.7 Earth masses.
- 165. 0.10 Earth masses.
- 166. Applying the refined third law of Kepler and neglecting the mass of the Earth compared with the Sun, we obtain 0.7 years.
- 167. Greater by a factor of 2.6
- 168. By 37%.
- 169. Greater by a factor of 2.5.
- 170. Greatest in January at perihelion and least in July, when the Earth is at aphelion. The ratio of these forces is 1.11.
- 171. 435.8 km/s.
- 172. $2^h 47.4^m$.
- 173. Four times greater than the distance to the Moon.
- 174. Cannot, since at distance $R = 6371$ km the orbital period is already 84.4 min.
- 175. Because the walls of the spacecraft and all objects

6.2 Structure of the Solar System

inside it have the same acceleration and do not press on the support. Their “fall” to the Earth occurs identically.

- 176. It is impossible at any altitude, because bodies in spaceflight are weightless.
- 177. 89.8 min.
- 178. No, it could not. The “horizon distance” for it was insufficient.
- 179. 16 revolutions.
- 180. For Earth – 7.83 km/s, 7.66 km/s, 7.61 km/s, 7.35 km/s.
For Mars – 3.12 km/s, 3.31 km/s, 3.42 km/s, 3.55 km/s.
- 181. 1330 km.
- 182. 329 km.
- 184. 1.67 km/s.
- 185. The launch vehicle, having a larger cross-section than the satellite, is more strongly slowed down by the atmosphere and therefore, while descending, moves faster.
- 186. On days 64 and 53 it is longer.
- 187. 8140 km and 91 020 km from the surface of Mars.
- 189. At perigee – 7.98 km/s; at apogee – 6.84 km/s.
- 190. Faster.
- 191. Its speed would be 4320 times greater than the speed of light.
- 192. At the equator.
- 193. 22 km.

6 Solutions

194. 70.5; 13°; 10.6°; 14.5°; 15°; 15°; 0°.
195. 232 m/s.
196. a) 465 m/s; b) 308 m/s.
197. At latitude φ , where $\cos \varphi = \frac{\cos 56^\circ}{2}$, i.e. $\varphi = 73.7^\circ$.
198. The winters of the northern hemisphere would become short and warm, and the summers long but colder than now. The alternation of day and night in summer and winter would be similar to the present.
199. Everywhere and always the duration of day would equal the duration of night, and the change of seasons would cease.
200. At an inclination of 45°.
201. About 30 km/s.
202. By a 1/5000 part of a second.
204. Degree measurements and determination of the force of gravity.
205. $a = R(2\pi/T)^2$, $a = 0.03391 \text{ m/s}^2$.
206. $g = \pi^2 l$, $g = 9.781 \text{ m/s}^2$.
207. $g = 9.809 \text{ m/s}^2$; $l = g/\pi^2$, $l = 993.85 \text{ mm}$.
208. Heating during the day would be greater; temperature fluctuations from day to night and from winter to summer would become greater.
209. See the answer to the previous problem.
210. 1 : 0.9171 : 0.3987.
211. $\cos(\varphi - 23.5^\circ) : \cos(\varphi + 23.5^\circ) = 3.4$.

6.2 Structure of the Solar System

- 212. 28.6; 2.5; 1.5; 1.0.
- 213. The influence of the inclination of the rays is greater than the influence of the distance to the Sun: $[\cos(\varphi - \varepsilon) : \cos(\varphi + \varepsilon)] : [(1 + e)^2 : (1 - e)^2] = 4.42 : 1.03 = 4.29$ times.
- 214. The acceleration of free fall at the equator exceeds the centrifugal acceleration of inertia by 289.4 times, therefore the Earth would have to rotate faster by $\sqrt{289.4} = 17.01$ times.
- 215. The difference in longitudes $\Delta\lambda = 105^\circ$ corresponds to an arc $l = (2\pi R \cos \varphi : 360) \cdot \Delta\lambda = 8251$ km.
- 216. 1'. By 900 times.
- 217. 42 000 000 km.
- 218. At 41° , at 2° .
- 219. 2.38".
- 220. 23.2".
- 221. 46.4".
- 222. 0.29". The concept of annual parallax is not applicable to planets.
- 223. 1.47".
- 224. 71 800 km.
- 225. 12 600 km.
- 226. Distance 60.3, and radius 0.272 of the Earth's radius; surface 0.0740 and volume 0.0201 relative to the surface and volume of the Earth.

6 Solutions

- 227. With an accuracy of 0.11% or 164 445 km.
- 228. 109 times, 1 391 000 km.
- 229. 4.2".
- 230. 34.6"; 59.5"; 62.8"; 17.0".
- 232. $(17.75'' : 8.80'') \cdot 5.431 \text{ au} = 10.95 \text{ au}$.
- 233. 6800 km.

6.3 Physical Nature of the Bodies of the Solar System

- 234. -30 km/s .
- 236. -20.7 km/s .
- 237. About $4 \mu\text{m}$.
- 238. The star is approaching us along the line of sight at 50 km/s .
- 241. The relative speed changes by 60 km/s , and $\Delta\lambda = \lambda v/c$, i.e., $\Delta\lambda = 1.2 \text{ \AA}$.
- 243. With an accuracy no worse than $0.15 \mu\text{m}$.
- 244. 54 km .
- 246. 5 km/s .
- 247. In a Martian day, Phobos's orbital period is $7^{\text{h}}39^{\text{m}}$. $24 : 24.6^{\text{h}} = 7.5^{\text{h}}$. Its diurnal motion among the stars is $360^\circ : (7.5 : 24) = 1151^\circ$. The interval between its culminations follows from the proportion $(360^\circ - 1151^\circ) : 360^\circ = 24^{\text{h}} : \tau$, hence

6.3 *Physical Nature of the Bodies of the Solar System*

$$\tau = 10^{\text{h}}55^{\text{m}}.$$

- 248. It would increase by the same factor as the increase of the area reflecting the light.
- 249. Because the day–night terminator on Venus is a semicircle, which we see at an angle as a half-ellipse.
- 250. At quadrature, when Earth looks like the Moon in first or last quarter.
- 252. By a fraction of 1 : 5000.
- 253. 5.5 Earth days.
- 254. By a factor of 82, and it would be 42.5° .
- 255. On Saturn's equator as a band crossing the sky through the zenith. At the poles, the ring is not visible.
- 256. By its displacement among the stars, noticeable over a few hours, when observed with the naked eye.
- 257. That they participate in the daily rotation of the sky. The same is shown by the measurement of their parallax.
- 258. Precisely it is impossible, because the tail gradually disappears.
- 260. It cannot, because the gas emitted from its nucleus when heated by the Sun becomes exhausted.
- 261. 18.5×10^7 years.
- 262. 0.34 and 4.10 astronomical units.

6 Solutions

- 263. Either because for some reason the self-luminosity of the comet weakened, or because the surface of the comet reflecting the Sun decreased, or due to the combined action of both causes.
- 264. Its orbital period is 5 years and 5 months.
- 265. From the Moon this would be completely unnoticeable.
- 266. Towards morning.
- 267. 10.34 astronomical units.
- 268. Its width is no less than the path travelled by the Earth during this time, i.e. $30 \text{ km/s} \times (37 \times 24 \times 3600) \text{ s} = 9.6 \times 10^7 \text{ km}$.
- 269. On the incomplete parallelism of their paths in the atmosphere.
- 271. 2.2 m.
- 272. By the fact that along this line the population density (probability of witnesses) is higher.
- 273. About 12.5 revolutions.
- 274. 27.32 d equal the sidereal month. Day and night equal half the synodic month, i.e. 14.76 d.
- 275. For an observer on the Moon, the Earth neither rises nor sets.
- 276. 2.43 km.
- 277. $\approx 3.5 \text{ km}$.
- 279. Yes, because the Earth, being usually a little higher or lower than the Sun, would be visible in the black

6.3 *Physical Nature of the Bodies of the Solar System*

sky of the Moon as a very thin crescent, with its convex side turned toward the Sun.

- 280. 80 times stronger than the lunar one.
- 281. 480,000 times.
- 282. The presence of shadows cast by irregularities of the lunar surface under oblique illumination by the Sun.
- 283. No. From the Moon the atmosphere surrounding the Sun is visible, which from the Earth is seen only during total solar eclipses.
- 284. No.
- 285. No, because these auroras occur in the atmosphere, which the Moon does not have.
- 286. The gravitational force on the Moon is 0.16 of the Earth's gravity.
- 287. 27.3 days.
- 288. 1.02 km/s.
- 289. No, because a solar day is shorter by 30 minutes than the interval between the culminations of the Moon
- 290. In winter at the greatest altitude.
- 291. $72^{\circ}8'$, $60^{\circ}36'$.
- 292. In the southeast.
- 293. In the first quarter.
- 294. In the evening.
- 295. The occultation is in the east.

6 Solutions

- 296. It is possible.
- 297. Near the zenith.
- 298. No, because the Moon has no magnetic field.
- 300. Western.
- 301. It cannot.
- 305. Smaller curvature of the inner part of the crescent, rapid change of phase, observation time, and the brownish colour of the eclipsed part of the Moon.
- 307. This is explained by refraction in the Earth's atmosphere
- 308. It can.
- 309. It can. It cannot, because the interval between the epochs of eclipses is only slightly shorter than half a year.
- 310. No, because there is polar night there.
- 311. Because the spots of light in the shadow of foliage are images of the Sun formed by tiny openings among the leaves, similar to a camera obscura.
- 313. No, because a decrease of the apparent angular diameter of the Moon by 10% is enough for the Sun not to be completely covered.
- 314. It can. It cannot.
- 315. 5 February 1981 and 31 July 1981.
- 316. The Moon is at apogee. Although at that time the Moon crosses the smaller diameter of the Earth's shadow cone, it moves more slowly and this effect

prevails.

317. They occurred in 1959 and in 1977; the next will be in 1995.

6.4 The Sun and Stars

325. Because the activity of the Sun has become higher.
326. $8.4''$.
327. 500 km.
328. ≈ 38 g.
329. ≈ 13.8 kg.
330. ≈ 12.6 tonne.
331. The amount of heat needed to melt the entire mass of ice is $Q_1 = 4\pi R^2 h \rho \lambda$, and the amount of heat received by the Earth from the Sun is $Q_2 = 4R^2 q t$, where R is the radius of the Earth, h is the thickness of the ice layer, ρ is its density, λ is the specific heat of fusion of ice, q is the solar constant, and t is the time it takes to melt the ice. $Q_1 = Q_2$, then $t = h\rho\lambda/q$, $t = 40$ min.
338. Taking into account the possible parallax error, the latter lies within the range from $0.012''$ to $0''$, and the distance within the range from 83 pc to infinity, i.e. one can only say that the star is at least 80 pc away.

6 Solutions

- 339. $3.4 \times 10^4 \text{ g/cm}^3$.
- 340. 8.8 ly.
- 341. 100.
- 343. By 6310 times.
- 344. $\pi R^2 : (\pi R^2 - \pi r^2) = 2$, $r : R = \sqrt{0.5}$, $r : R = 0.7$.
- 345. By 17 times.
- 346. $2.7 \text{ pc} = 8.8 \text{ ly} = 558\,000 \text{ au} = 8.3 \times 10^{13} \text{ km}$ (for Sirius).
- 347. By 10500 times.
- 348. The star in Dorado is brighter by 500 000 times.
- 349. By 14000 times.
- 350. 11.6 and 13.4.
- 351. 0.0020 and 0.00038.
- 352. Sirius is 10200 times brighter than its companion.
- 353. +1.27.
- 354. -4.0.
- 355. $5 \text{ pc} = 16.3 \text{ ly}$.
- 356. The parallax lies within $0.306''$ to $0.318''$; therefore, the distance is within 3.26 pc to 3.14 pc, i.e. known to an accuracy of a few percent.
- 358. 7.5 solar masses.
- 359. 1.5 solar masses.
- 360. 2.1 solar masses.
- 361. 5.5 solar masses.
- 362. Will decrease by a factor of 1.7.
- 363. $0.75''$. Possible.

6.5 Structure of the Universe

- 364. 3.68713^d .
- 365. Amplitude from 5.5 to 13.5 in stellar magnitude, period 400.
- 366. Period 5.3^d , amplitude from 3.7 to 4.1. This is Delta Cephei.
- 367. The star has irregular variations in brightness. This is χ Cygni.
- 368. 240 pc.
- 369. 166 pc.
- 370. 17.4.
- 371. By a factor of 480.
- 372. By a factor of 15.8.
- 373. 30 June of the next year; 9 August a year later; 17 September another year later.
- 374. By a factor of 3.4.
- 375. By a factor of 100.
- 376. Answer to the last question: from a distance of 1.8 pc, i.e., farther than the nearest stars.
- 377. 700 km/s.

6.5 Structure of the Universe

- 384. $V_r = V \cos \theta$, $v_r = 43 \text{ km/s}$, $V_\tau = V \sin \theta$, $V_\tau = 25 \text{ km/s}$.
- 385. 4.1 au.

6 Solutions

- 386. They increase in the region of the constellations Lyra and Hercules and decrease in the opposite part of the sky.
- 387. In 97000 years.
- 388. The velocity of the Earth relative to the star at these two moments is opposite, since the moments are separated by half a year, and therefore it is equal to twice the Earth's velocity projected onto the line of sight directed to the star. Thus, this velocity will vary from 0 to ± 60 km/s depending on the location of the star on the ecliptic.
- 389. The constellation Hercules.
- 390. 57 km/s.
- 391. 46° .
- 392. 25 km/s at an angle of 31° .
- 393. In 400 000 years.
- 394. 16 km/s.
- 395. +18 km/s and 14 km/s.
- 396. $0.14''$.
- 397. 90 km/s.
- 398. 17 km/s.
- 399. About one star per pc^3 .
- 400. 55 000 au.
- 401. 3 400 000 years.
- 402. 40 000 pc.
- 403. 35 pc.

6.5 Structure of the Universe

- 404. 10 500 pc.
- 405. 37 pc; -9.2.
- 406. 240 000 pc. Diameter 13 500 light years.
- 407. 100 megaparsecs.
- 408. 14 000 parsecs.
- 409. By a factor of 15.86, since the corresponding numbers of stars relate as the volumes of spheres whose radii are in the ratio 2.512:1.
- 410. A faint star farther away.
- 414. 0.006 mag.
- 415. 6.3×10^7 pc, or about 2×10^8 ly.
- 416. Taking this into account, their diameters become smaller than previously assumed, because interstellar extinction reduces the apparent brightness of Cepheids in galaxies; we therefore overestimated their distances, and likewise assigned the observed galaxy diameters to too large a distance.
- 418. $m = a - 5 \log d$, where a is a constant.
- 419. The relationship on the graph of the dependence of m on $\log d$ will be represented by a straight line that is more inclined to the axis on which we will plot $\log d$ than in the previous problem.

FINISH